**VOLUME 2 (1956/57) No. 3**

# NUCLEAR PHYSICS

**MONTHLY JOURNAL DEVOTED TO THE EXPERIMENTAL AND THEORETICAL STUDY OF ATOMIC NUCLEI, NUCLEAR FIELDS AND THE FUNDAMENTAL ASPECTS OF COSMIC RADIATION**

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NUCLEAR PHYSICS is published in monthly issues. The subscription price of a volume of 700 pages is fl 57.00; 114s; $ 15.00. Postfree.

SUBSCRIPTIONS should be sent to the publishers, NORTH-HOLLAND PUBLISHING COMPANY, P.O.Box 103, Amsterdam or to any subscription-agent. Subscriptions for U.S.A. are to be sent exclusively to INTERSCIENCE PUBLISHERS, INC., 250 Fifth Avenue, New York 1, N.Y.

Printed in the Netherlands.

*Nuclear Physics* **2** (1956/57) 193–225; *North-Holland Publishing Co., Amsterdam*

# ENERGY LEVELS IN $Pb^{205}$ FROM THE DECAY OF $Bi^{205}$

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Received 5 July 1956

**Abstract:** The energies, spins and parities of 9 excited states in $Pb^{205}$ and the probable energies of 8 additional states have been established by studies of 14.5-day $Bi^{205}$ decay. The experiments include precision energy measurements on conversion lines in permanent magnet and double focusing spectrometers; measurements of coincidences between pairs of conversion lines in a double long lens spectrometer, and coincidences between high energy gamma rays and low energy conversion lines in the same spectrometer adapted for the purpose; measurements of relative gamma intensities in a scintillation spectrometer. Positons were discovered with a maximum energy of 925 keV, establishing the total decay energy of $Bi^{205}$ as 2.65 MeV. The proposed decay scheme contains 28 of the observed transitions and is discussed in the light of the shell model.

## 1. Introduction

This investigation of the levels in $Pb^{205}$ is part of a general program of investigating the nuclei in the region of the double magic ${}^{126}_{82}Pb^{208}$. This program was initiated by the work of D. Alburger [1]) (1952) and continued by several teams at the Nobel Institute of Physics and at the Gustaf Werner Institute for Nuclear Chemistry (1954).

The nucleus $Pb^{205}$ differs from the double-closed shell in $Pb^{208}$ by three neutrons. Some striking features of the level scheme of $Pb^{206}$ (2 "holes") could be understood in terms of the shell model knowing the levels of $Pb^{207}$ (1 "hole"). It was hoped that an experimental investigation of the levels in $Pb^{205}$ would give a further check to theory and lead to a better understanding of the level systematics in the region of the double magic ${}^{126}_{82}Pb^{208}$.

Before the commencement of this work, the known facts relevant to the levels of $Pb^{205}$ were the following:

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[††] On leave of absence from the University of Melbourne.

Karraker and Templeton [2]) found that $Bi^{205}$ decays by electron capture to $Pb^{205}$ with a half-life of 14.5 days. They also report gamma rays of the following energies: 431, 527, 550, 746 and 1840 keV.

Alburger [1]), however, did not find any of these gamma transitions but has reported conversion lines corresponding to the energies 284.4, 703.7, 911.6, 988.6, 1044.6, 1074.1, 1189.8, 1615.4, 1766.3, 1777.8 and 1864.2 keV.

In α-decay of $Po^{209}$ Dagget and Grove [3]) found gamma rays of energies 260, 570 and 865 keV which were assumed to be emitted from levels of energies 570, 840 and 865 keV.

Unpublished measurements [4]) at the Radiation Laboratory (Berkeley) indicate, however, that only one gamma ray of energy 260 keV is fed by α-decay of $Po^{209}$.

Harvey [5]) measured triton groups from a (d, t) reaction on lead and suggests levels at 270, 840 and 1790 keV.

In the present work about 40 gamma rays were found, including 10 of the 11 gamma rays observed by Alburger (the 1074.1 keV gamma ray was shown to be non-existent). The gamma rays reported by Karraker and Templeton were not observed. Two gamma rays (260.5 and 571.0 keV) observed in our experiments have energies close to those observed by Dagget and Grove. Coincidence experiments, however, showed that their gamma rays cannot originate from $Pb^{205}$.

## 2. Experimental Procedures

The levels of $Pb^{205}$ have been studied in the decay of $Bi^{205}$, which was produced by bombarding natural lead with 25 MeV deuterons in the 225-cm cyclotron of the Nobel Institute. After chemical separation, Bi-sources were prepared for investigations in different spectrometers.

Accurate values of energies and relative conversion line intensities were obtained from measurements in a double focusing beta spectrometer [6]). Energy determinations were also made in a permanent magnet spectrometer for photographic recording [7]).

Positons were studied in an intermediate-image beta spectrometer [8]) and a double focusing spectrometer [6]).

A single-channel scintillation spectrometer was used in order to estimate relative gamma intensities and to search for high energy gamma rays.

Electron-electron and electron-gamma coincidence measurements were performed in a double long lens spectrometer [9]) at the Physics Department of the University of Uppsala.

The permanent magnet spectrometer was found to be ideally suited for a general investigation of a complex conversion spectrum. It covers a wide energy region and has good resolving power (1 : 1000 in the slit setting used). The integrating property of a photographic detection is particularly valuable for investigating long half-life isotopes. On the other hand, the double focusing spectrometer, while possessing comparable resolving power, permits more accurate intensity measurements to be made. Small specific activity can be used in the double focusing spectrometer, because the activity can be spread out over a large area ($2 \times 15$ mm$^2$).

### 2.1. SOURCE PREPARATION

$Bi^{205}$ was produced in a $Pb^{206}$ (d, 3n) reaction by bombarding natural lead with 25 MeV deuterons (the current limited to 10 $\mu$A because of cooling difficulties). The threshold for the (d, 3n) reaction is about 16 MeV [1]). The abundances of the stable lead isotopes $Pb^{208}$, $Pb^{207}$, $Pb^{206}$ and $Pb^{204}$ are 52, 23, 24 and 1.5 % respectively. Therefore the 20 y $Bi^{207}$ and the 6 d $Bi^{206}$ are unavoidably produced in the same bombardment. The total lead activity immediately after a bombardment of about 400 $\mu$Ah was about 1 r/h at a distance of 40 cm. Two kinds of sources (denoted A and B sources) were used in the experiments.

*A sources:* After an irradiation ($\approx$ 200 $\mu$Ah) of commercial lead the Bi was chemically separated by a method described by Alburger and Friedlander [10]). These sources were comparatively thick, partly because of the presence of stable Bi in the lead, and were used mainly for studying conversion electrons of high energies and for measurements in a scintillation spectrometer.

*B sources:* After a later irradiation ($\approx$ 400 $\mu$Ah) of spectroscopically pure lead the separation method above was modified by use of column chemistry [11]). It was also found to be very important to use extremely pure HCl, which for this reason was purified by use of ion exchange columns. These sources were sufficiently thin to enable the study also of very low energy electrons.

Beta spectrometer sources for the double focusing spectrometer were prepared by electroplating the Bi from 1 $N$ $H_2SO_4$-solutions onto Cu foils of thickness 5 mg/cm$^2$. The surface of each foil was masked off with nail polish, except for an area corresponding to the desired source shape (usually about $2 \times 15$ mm$^2$). The applied voltage was a few volts, and a Pt stirrer was used as anode. About 50 % of the activity was transferred to the source in times varying from half an hour to several hours. A source for the permanent magnet spectrometer was prepared in the same way, by electroplating onto a Pt wire (0.2 mm diameter) the back of which was masked off with nail-polish.

### 2.2. CONVERSION ELECTRON SPECTRUM

For an investigation of the conversion electrons in the energy region 90—620 keV a B source was used. Fig. 1 shows a small part of the spectrum as recorded in the double focusing spectrometer with a relative half-width of 0.14 %. The measurements were made about one month after the bombardment, in order to reduce the intensity of the strong lines of the 6 d Bi$^{206}$. Fig. 2 shows the conversion lines in the energy region 940—1870 keV measured with an A source and with 0.35 % resolution in the double focusing spectrometer. The energy region 30—1000 keV was investigated with a B source in a permanent magnet spectrometer, (see fig. 3). For energy calibration the K conversion lines of the 569.7 and 1063.9 keV gamma rays of Bi$^{207}$, as well as the K conversion line of the 662 keV gamma ray of Cs$^{137}$ were used. Figures 1—3 show the importance of using good resolution when a complex decay is investigated. They also illustrate the difficulties introduced in our measurements because of the presence of Bi$^{206}$ and Bi$^{207}$ lines. The energy values of the Bi$^{206}$ lines agree in all cases with those given by Alburger [1]) to better than 1 : 1000 and usually to better than 3 : 10 000. The source used in the permanent magnet spectrometer for the exposure shown in fig. 3 was about 10 times weaker than that used in the double focusing spectrometer (both of them B sources), (cf. fig. 1). Nevertheless, after one week's exposure, more lines could be detected in the photographic film than by measurements in the double focusing spectrometer (especially in the region of maximum sensitivity of the Ilford Selochrome film). This shows the advantage of the integrating property of the photographic detector.

The conversion lines were identified by their half-lives (14.5 d), obtained from two continuous runs (and more in some regions) in the double focusing spectrometer and from several successive exposures in the permanent magnet spectrometer.

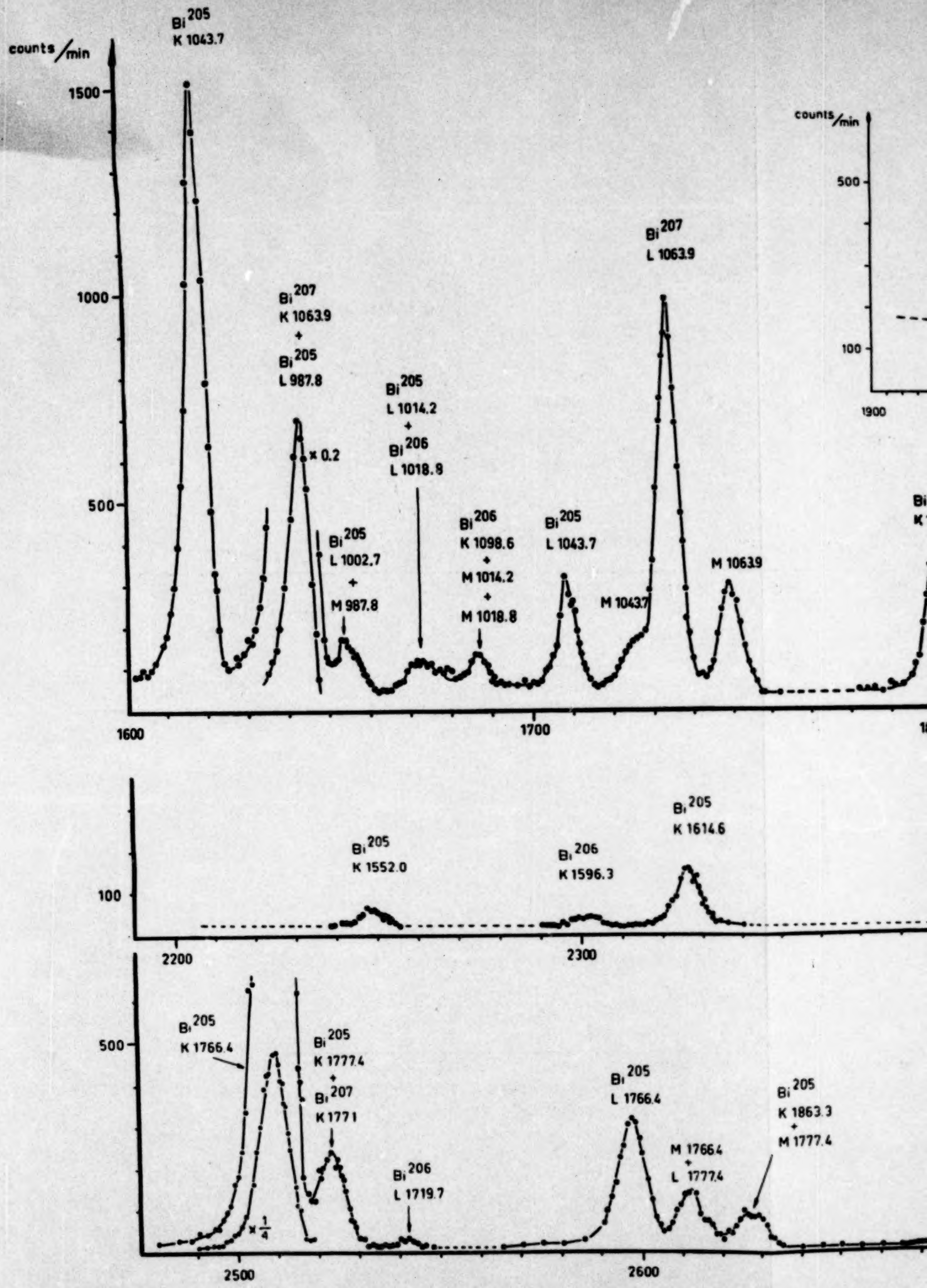
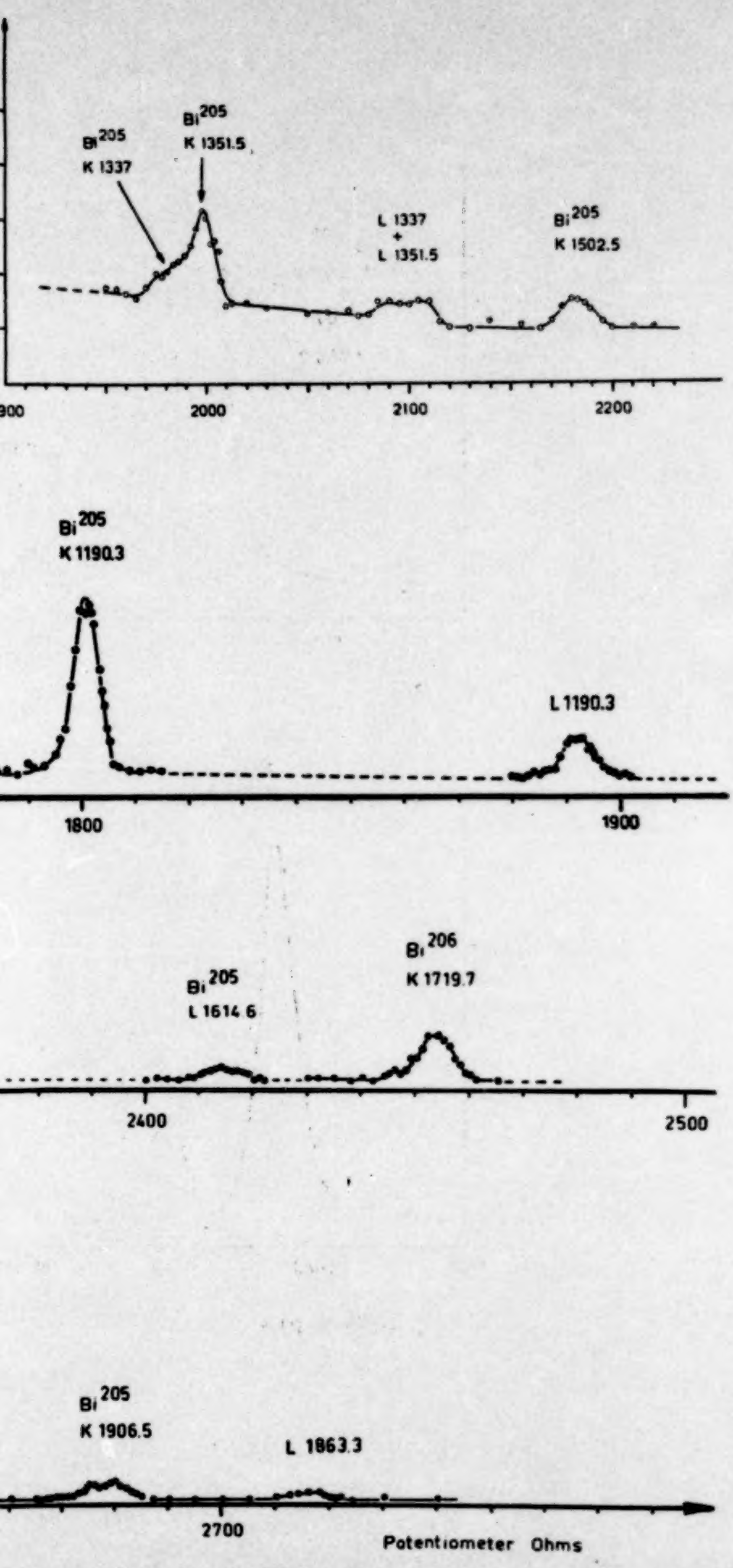


Fig. 2. High energy part of the conversion electron spectrum measured with 0.35 %
The inset shows a small part of the spectrum measured with



0.35 % resolution in a double focusing spectrometer.
d with 0.8 % resolution.

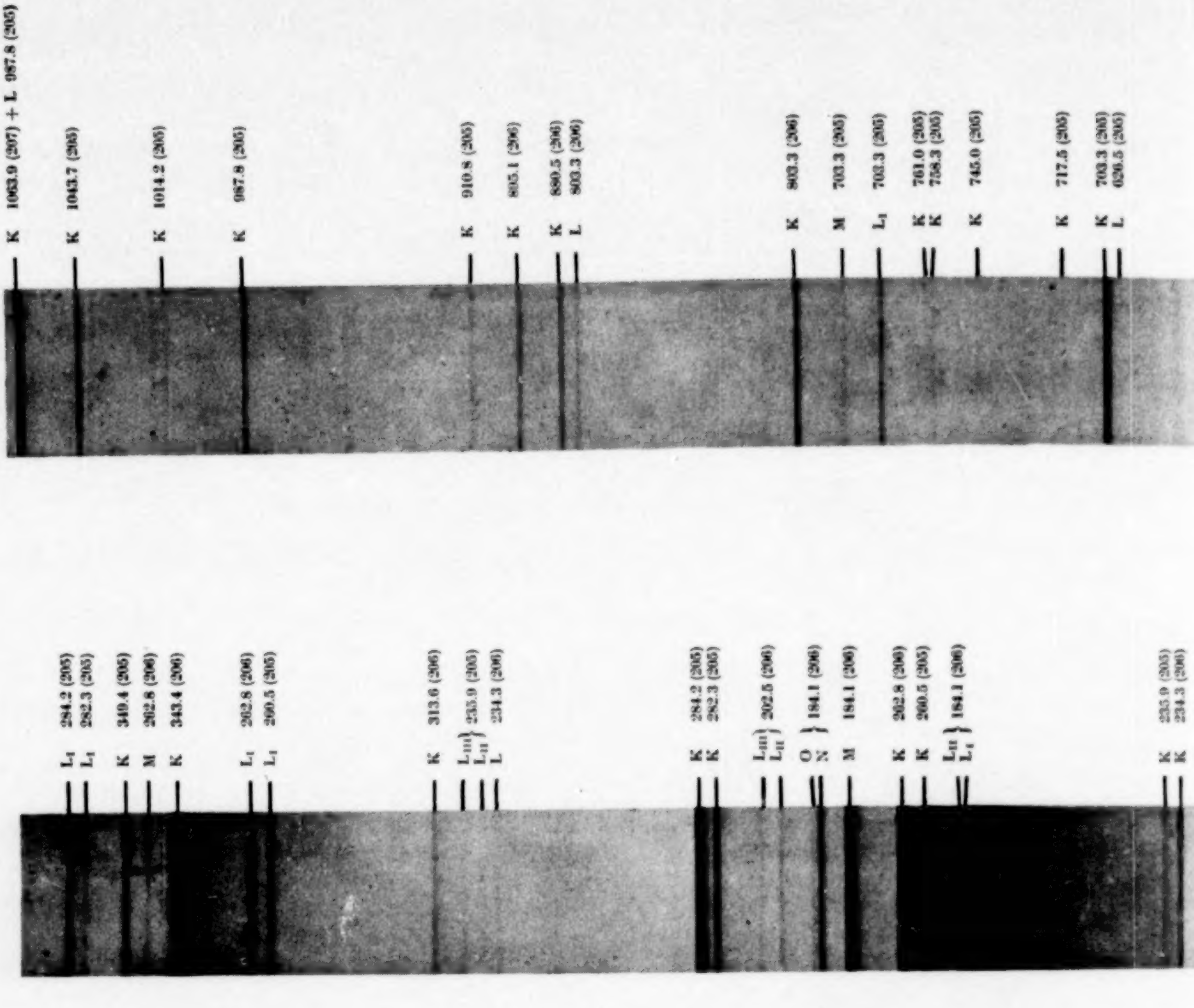
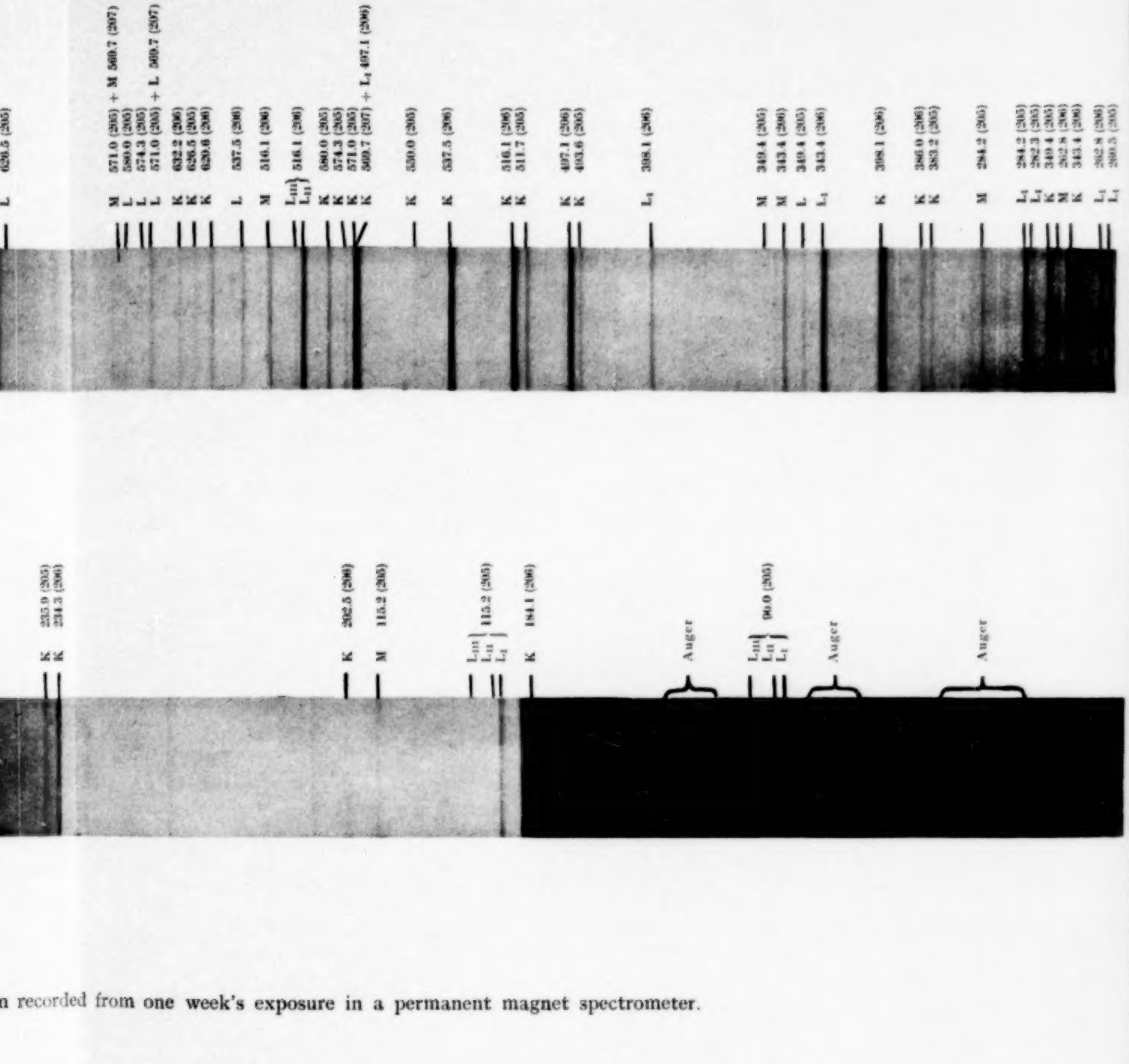


Fig. 3. Low energy part of the conversion electron spectrum recor



n recorded from one week's exposure in a permanent magnet spectrometer.

The information obtained from the investigations of the conversion electrons is summarised in tables I, II and III. Table I includes all gamma rays whose energies are considered to be certain. In the region below 700 keV the energy values were taken from the

TABLE I

Gamma transitions in $Pb^{205}$

| $\gamma$-ray energy (keV) | K line intensity (K 703.3 = 100) | $\gamma$-ray intensity (in percent per disintegration) | Transition intensity (in percent per disintegration) adopted from K line intensity |
|---|---|---|---|
| 115.2 | 10 † | | |
| 235.9 | 8 | | |
| 260.5 | 110 | | |
| 282.3 | 41 | 1.2 | 0.20 |
| 284.2 | 160 | | 0.80 |
| 349.4 | 28 | | 0.23 |
| 383.2 | 6 | | |
| 493.6 | 11 | | |
| 511.7 | 22 | | 0.55 |
| 550.0 | 6.3 | | |
| 571.0 | 74 | 2.3 | 2.1 |
| 580.0 | 24 | | |
| 626.5 | 6.7 | | |
| 688.9 * | 1 | | |
| 703.3 | 100.0 | 14.5 | 14.5 |
| 745.0 | 2.1 | | |
| 758.3 | 3.1 | | |
| 761.0 | 2.0 | | |
| 910.8 | 9.8 | 3.3 | 2.4 |
| 987.8 | 27.5 | 14 | 6.6 |
| 1002.7 | 2.1 | | |
| 1014.2 | 4.9 | | |
| 1043.7 | 32.5 | 2.2 | 3.7 |
| 1190.3 | 7.6 | $<2$ | |
| 1337 * | 0.3 | | |
| 1351.5 | 1.1 | | |
| 1502.5 * | 0.4 | | |
| 1552.0 | 0.8 | | |
| 1614.6 | 3.1 | $<3$ | |
| 1766.4 | 43.6 | 52 | 37 |
| 1777.4 | 5.2 | | 16 |
| 1863.3 | 2.0 | 9 | 5.4 |
| 1906.5 | 0.6 | | |
| $2600 \pm 50$ | $<0.5$ †† | 0.3 | |

† L line intensity.

†† Found only in scintillation spectrum.

* Accurate to 1 : 1000. The other energies are accurate to 5 : 10 000.

permanent magnet photographic film. The energy values in the region above 700 keV were taken as weighted averages from two continuous runs in the double focusing spectrometer. All energies are accurate to better than 5 : 10 000, except for the weak lines marked with a star in table I, which are accurate to 1 : 1000. In cases where the multipolarity assignment could be made (cf. table III), the transition intensities given in column 4 of table I were calculated from the theoretical K conversion coefficients [12]) and the relative K conversion intensities.

In table II conversion lines are given for which the assignment is not certain. The energy accuracy of these lines is generally about 1 : 1000.

TABLE II

Weak lines in $Pb^{205}$

| Electron energy (keV) | Intensity † (K 703.3 = 100) | Possible assignement |
|---|---|---|
| 13.17 | | |
| 22.55 | | K 110.6 |
| 22.81 | | |
| 25.30 | | K 113.3 |
| 25.57 | | |
| 26.06 | | |
| 62.17 | <1 | $L_I$ 78.1 |
| 65.37 | <1 | $L_{III}$ 78.4 |
| 74.18 | 10 | $L_I$ 90.0 or K 162.2 |
| 74.70 | 1 | $L_{II}$ 89.9 |
| 77.08 | 5 | $L_{III}$ 90.1 |
| 97.30 | 10 | $L_I$ 113.2 or $L_{III}$ 110.3 |
| 106.0 | <1 | M 110 |
| 107.4 | <1 | M 110 |
| 108.4 | <1 | N 110 |
| 117.8 | 4 | L 133 |
| 133.0 | 5 | K 221.0 or $L_{III}$ 146.0 |
| 142.6 | 2 | M 146 |
| 149.2 | 1 | K 237.2 |
| 273.6 | 1 | $L_{III}$ 286.6 |
| 205.4 | <1 | $L_I$ 221.3 |
| 209.7 | <1 | |
| 365.5 | <1 | L 383.2 |
| 486.3 | 9 | K 574.3 |
| 629.5 | 2 | K 717.5 |

† The intensities were taken in most cases from the permanent magnet spectrometer films and should be considered as rough estimates. Below 30 keV the sensitivity of the film changes very rapidly with electron energy, so that an estimate of the intensity is very difficult.

In table III only those gamma rays are listed for which multipolarity information is available. The multipolarity assignments were made on the basis of K/L ratios [13]), conversion in L subshells [13]) and absolute K conversion coefficients [12]). The latter were calculated by comparing the gamma ray intensities in the scintillation spectrum with the K conversion line intensities.

TABLE III

Multipolarity assignments of transitions in $Pb^{205}$

| $\gamma$-ray energy (keV) | Internal conversion data | Multipolarity assignment |
|---|---|---|
| 260.5 | $K:(L_I+L_{II}) = 3.4\pm0.6$; $L_I:L_{II} = 5^{+4}_{-2}$ | M2 |
| 282.3 | $K:(L_I+L_{II}) = 5.5\pm1.0$ | M1 |
| 284.2 | $K:(L_I+L_{II}):M_I = (5.3\pm1):1:(0.20\pm0.03)$;<br>$L_I:L_{II} = 6\pm2$; no $L_{III}$ | M1 |
| 349.4 | $K:L = 5.9\pm0.5$; $M_I$ conversion only; $\alpha_K > 3\times10^{-2}$ | M1 |
| 511.7 | $K:L = 6.0\pm1.5$; $\alpha_K > 3\times10^{-2}$ | M1 |
| 571.0 | $K:L = 5.5\pm0.5$; $\alpha_K = (5\pm2)\times10^{-2}$ | M1 |
| 580.0 | $K:L = 7.0\pm2.5$; $\alpha_K > 3\times10^{-2}$ | M1 |
| 703.3 | $K:L:M = (4.2\pm0.1):1:(0.35\pm0.05)$;<br>$\alpha_K = (1.0\pm0.3)\times10^{-2}$ | E2 |
| 910.8 | $K:L = 4.0\pm0.7$ | E2 |
| 987.8 | $K:L = 4.0\pm0.5$; $\alpha_K = (2.5^{+2}_{-1})\times10^{-3}$ | E2 (or E1?) |
| 1043.7 | $K:L:M = (5.7\pm0.2):1:(0.33\pm0.05)$;<br>$\alpha_K = (2\pm1)\times10^{-2}$ | M1 |
| 1190.3 | $K:L = 3.8\pm0.5$ | M4, M3 or E2 |
| 1614.6 | $K:L = 7.0\pm1.5$ | |
| 1766.4 | $K:L = 6.0\pm0.3$ | E2 † |
| 1863.3 | | E1†† |

† Best fit between K line intensity and gamma ray intensity of the 1766.4 and 1777.4 keV transitions is obtained by assuming multipolarities E2 and E1 respectively.

†† Best fit between K line intensity and gamma ray intensity of the 1863.3 and 1906.5 keV transitions is obtained by assuming the multipolarity E1 for both.

### 2.3. POSITON SPECTRUM

Since $Bi^{205}$ decays by electron capture, it was natural to search for positons, although positon emitters had not been found in this *Z* region at the time when this investigation was performed.

Positons were found with an A source in the intermediate-image spectrometer, in which helical baffles stopped the electrons. The positon spectrum was found to decay with a half-life of about

14 days and is therefore associated with the decay of $Bi^{205}$. The maximum positon energy as obtained from a Fermi plot was 970±50 keV. However, the background of gamma rays and scattered electrons was high compared to the positon intensity. For this reason the positon spectrum was later remeasured in the double focusing spectrometer with a B source. The background was then considerably decreased because of the use of a coincidence counter and because of the large dimensions of the spectrometer, which enables efficient shielding. The positon spectrum and a Fermi analysis are shown in fig. 4. The positon energy obtained in this measurement was

$$E_{\beta^+} = 925 \pm 30 \text{ keV}.$$

It is difficult to come to conclusions about the shape of the positon spectrum because of poor statistics. Positons are also expected due to internal pair formation of the strong 1766.4 and 1777.4 keV gamma rays. By assuming a pair conversion coefficient of $2 \times 10^{-4}$ [14]) for the strong gamma rays, the intensity of these positons is estimated at 10 % of the total positon intensity observed. The deviation from a straight line of the Fermi plot in the region 700—800 keV might be due to these positons from internal pair formation. However, the possibility that the positon spectrum has a forbidden shape cannot be excluded.

The positon spectrum below 100 keV is influenced by the thick copper backing used (about 5 mg/cm$^2$). It is therefore not possible to decide from the Fermi plot of fig. 4, if the excess of low energy positons is also due to a low energy component.

Coincidence experiments indicated (see section 2.7) that positons are in coincidence with the 703.3 keV gamma ray (cf. decay scheme in fig. 8).

### 2.4. SCINTILLATION SPECTRUM

Scintillation spectra were measured with a single-channel analyser 11 and 8.5 weeks after the irradiation and are shown in figures 5 and 6 respectively. The resolution with the NaI(Tl)-crystal was 8 % for the 662 keV gamma ray of $Cs^{137}$. The relative gamma intensities of the strongest gamma rays (see table I, column 3) were calculated from the known crystal efficiencies. A gamma ray of energy 2.60±0.05 MeV, the conversion lines of which were not seen in the double focusing spectrometer, was found, as shown in fig. 6. Any other gamma ray in the decay of $Bi^{205}$ with an energy

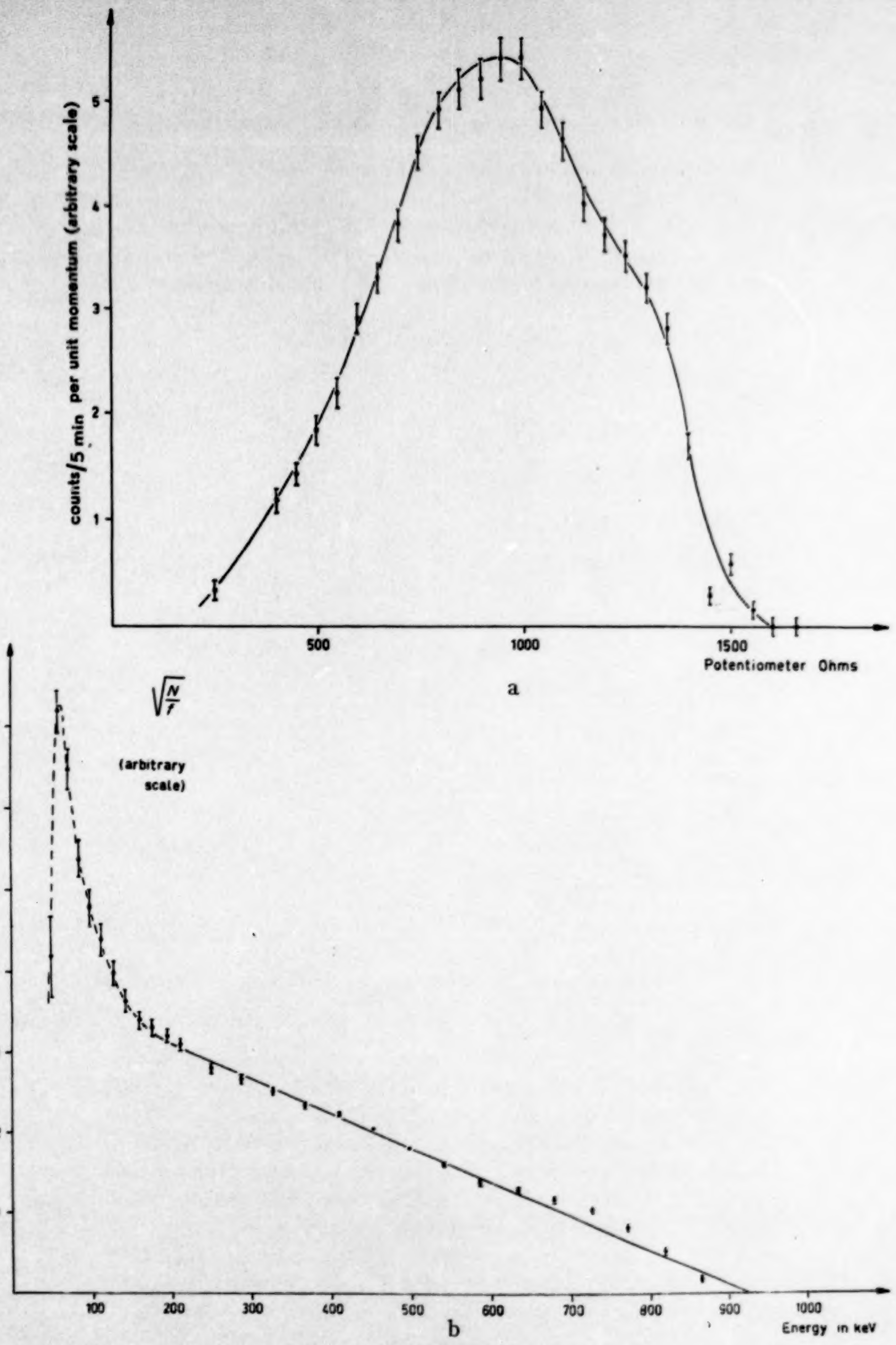


Fig. 4. a) Positon spectrum measured in a double focusing spectrometer. The background (10 % of the peak intensity) was subtracted. b) Fermi plot of the positon spectrum shown in a). Allowed shape was assumed. Screening corrections have been made. The errors shown are standard deviations.

larger than 2.6 MeV must have an intensity which is less than 0.1 (cf. table I, column 3).

As will be discussed later (section 3.1), positon measurements suggest that the decay energy of $Bi^{205} \rightarrow Pb^{205}$ is 2.65 MeV. In order to check that there is no level higher than 2.6 MeV which can be reached in the decay of $Bi^{205}$, an investigation was made with a hollow crystal (1.5″ Harshaw NaI(Tl)). The ratio between the areas of the 2.6 MeV and 1766.4 keV photo peaks changed from about 1 : 400 outside the crystal to about 1 : 20 when the source was placed inside the crystal. This shows that there is a considerable coincidence effect of gamma rays, the energies of which sum up to about 2.6 MeV. No sum-up peak with an energy larger than 2.6 MeV was found, which confirms that the decay energy of $Bi^{205}$ cannot be much larger than 2.6 MeV. The complexity of the

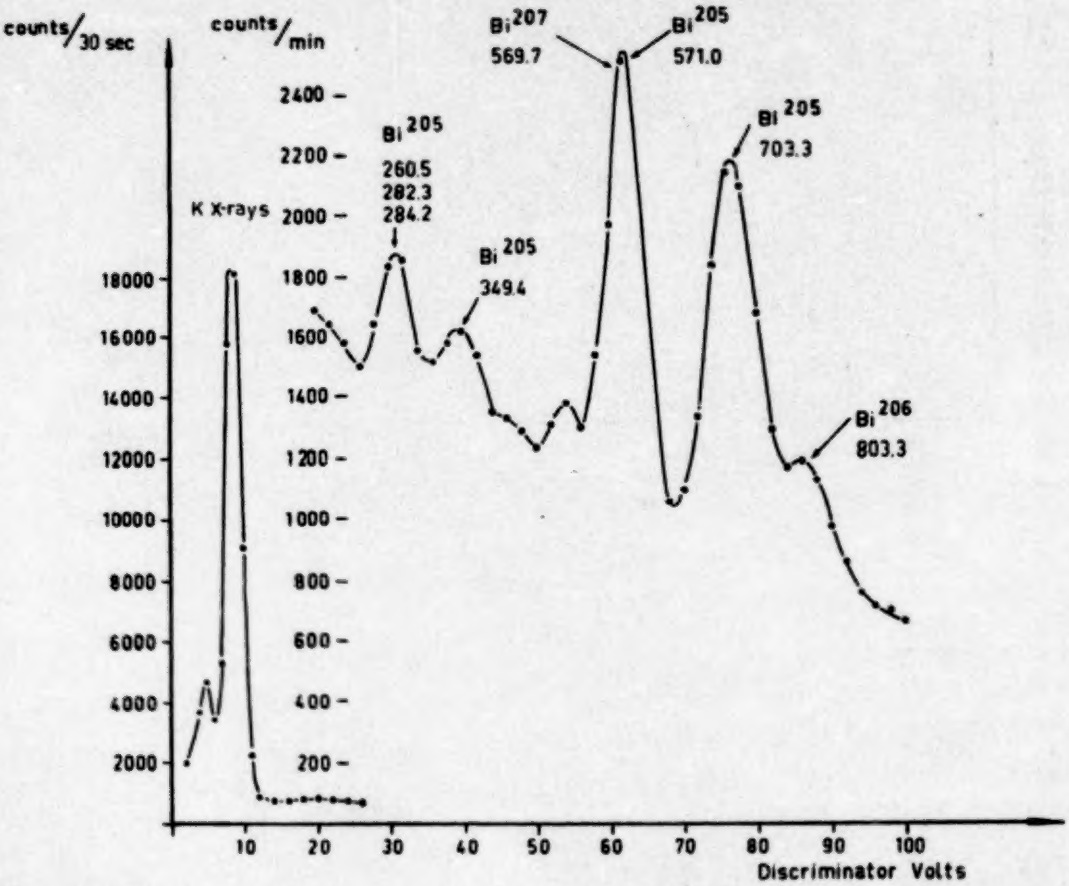


Fig. 5. Scintillation spectrum (low energy part) measured with a single-channel analyser 11 weeks after bombardment.

scintillation spectrum obtained with the hollow crystal makes an analysis of sum-up peaks of lower energies than 2.6 MeV difficult.

### 2.5. SEARCH FOR A METASTABLE STATE

The search for an M4 transition from a metastable level in $Pb^{205}$ has previously been reported by us [11]). Several M4 transitions are

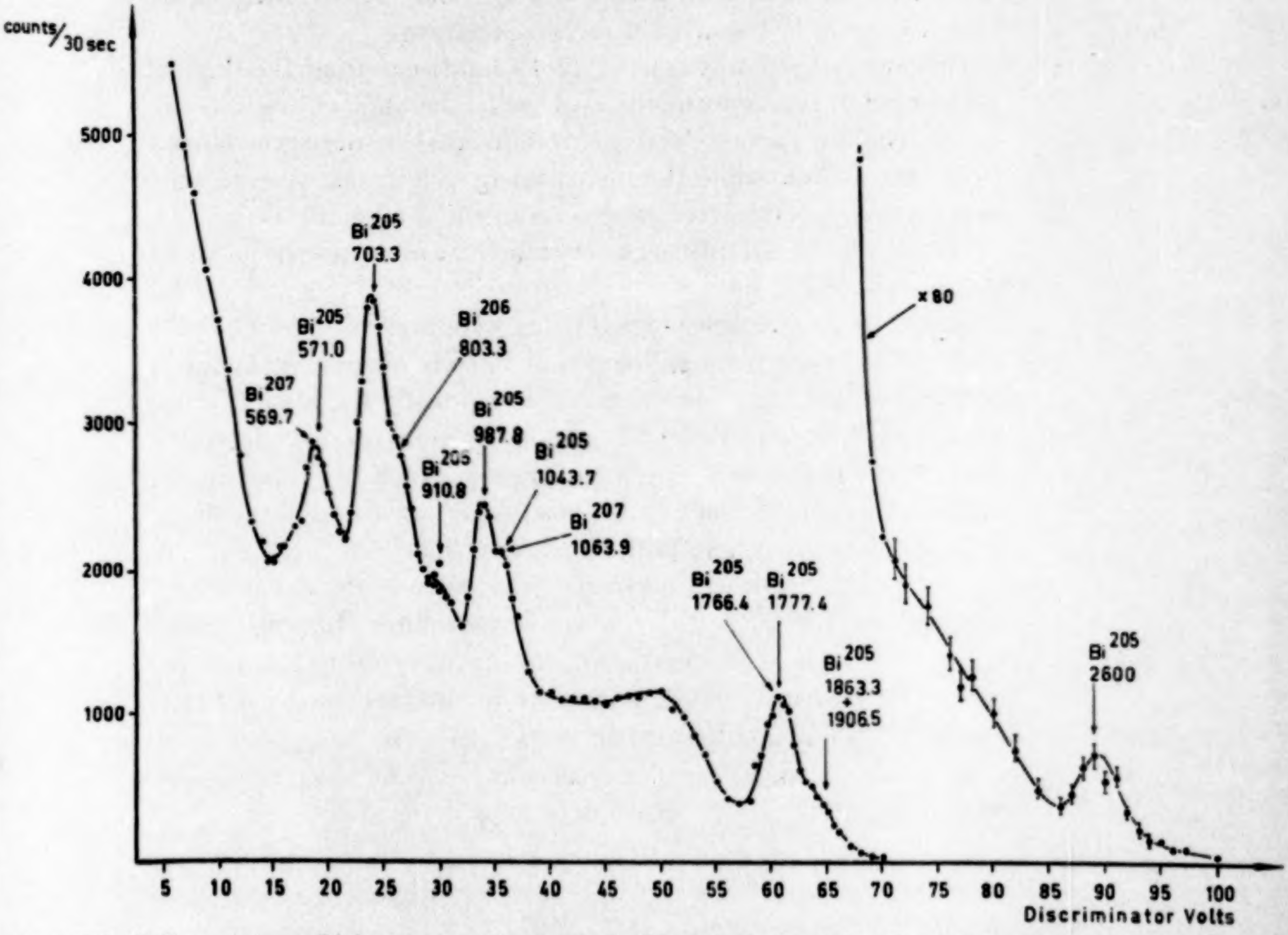


Fig. 6. Scintillation spectrum (high energy part) measured with a single-channel analyser 8.5 weeks after bombardment.

now known from $Pb^{197m}$ to $Pb^{207m}$ exhibiting a systematic variation of energy and half-life as a function of the neutron number. From the systematic behaviour of these isomers, an energy of 980 keV and a half-life of 1.5 sec is expected [11]) for an $i_{13/2} \rightarrow f_{5/2}$ transition in $Pb^{205}$. However, our milking experiments did not show up the corresponding M4 transition. Furthermore, the expected activity has not been seen by Bendel *et al.* [15]) in (γ, n) reactions on lead.

### 2.6. ELECTRON-ELECTRON COINCIDENCE MEASUREMENTS

The large number of conversion lines found in the decay of $Bi^{205}$ made coincidence experiments indispensible for unravelling the decay scheme.

The experiments were made in a double long lens spectrometer designed specially for $e^-$—$e^-$ coincidence work [9]). The electrons were detected with anthracene crystals, and the resulting pulses were fed to a fast-slow coincidence apparatus.

The source used was a piece ($2 \times 3$ mm$^2$) cut from the original $2 \times 15$ mm$^2$ B source formerly used in the double focusing spectrometer. The low energy electrons were focused in the spectrometer facing the source, while the higher energy electrons were focused in the other spectrometer after passing the 5 mg/cm$^2$ Cu backing. (This passage did not influence very much the intensities or positions of the conversion lines.)

The $e^-$—$e^-$ coincidence experiments were performed two months after the (second) bombardment, but in spite of that, each pair of conversion lines had to be examined individually for possible interference of $Bi^{206}$ (and $Bi^{207}$). (It was not possible to investigate the pair 571.0—1043.7 keV in the coincidence experiments because of interference from the 569.7 and 1063.9 keV transitions in the decay of $Bi^{207}$.) In cases where both conversion lines were well resolved, the coincidence rate was measured only when both channels were focussed on the peaks of the conversion lines. In some cases, however, the whole conversion line in coincidence with another line had to be measured. Most pairs were investigated at least twice, and the reproducibility was very good.

Relations of significant interest for the coincidence measurements are

$$C_g = N_1 \delta_2 \varkappa_2 \omega_2 \varepsilon_2 \varepsilon_c \qquad (1)$$

$$C_{ch} = 2\tau N_1 N_2 \qquad (2)$$

where the indices 1 and 2 refer to two nuclear transitions, No. 1 (detected in spectrometer 1) feeding (directly or indirectly) a level from which No. 2 (detected in spectrometer 2) takes place, and further

$C_g$ = number of genuine coincidences per unit time,
$C_{ch}$ = number of chance coincidences per unit time,
$N_1$ = counting rate of conversion line 1 in channel 1,
$N_2$ = counting rate of conversion line 2 in channel 2,

$\delta_2$ = fraction of transition 1 in coincidence with transition 2,
$\varkappa_2$ = K conversion probability [†††] of transition 2,
$\omega_2$ = solid angle in spectrometer 2,
$\varepsilon_2$ = efficiency of the detector (anthracene crystal) in spectrometer 2,
$\varepsilon_c$ = efficiency of the coincidence circuit $\approx 1$,
$\tau$ = resolving time of the coincidence circuit $= 2\times 10^{-7}$ s.

In formula (1) above angular correlation effects have been

TABLE IV

Electron-electron coincidences in the decay of $Bi^{205}$

| Pair number | Energy (in keV) of gamma-ray, conversion electron of which is focused in: channel 1 | channel 2 | $C_g$ counts/min | Assumed value of $\varkappa_2$ | Calculated value of $\delta_2$ (%) |
|---|---|---|---|---|---|
| 1 | 260.5 | 349.4 | $4.0 \pm 0.3$ | 0.18 (M1) | $7.4 \pm 0.7$ |
| 2 | 260.5 | 282.3 + 284.2 | $2.5 \pm 0.3$ | 0.28 (M1) | $2.0 \pm 0.2$ |
| 3 | 260.5 | 571.0 | $2.3 \pm 0.3$ | 0.053 (M1) | $8.3 \pm 1.0$ |
| 4 | 282.3 | 284.2 | $0.37 \pm 0.15$ | 0.28 (M1) | $2.5 \pm 0.8$ |
| 5 | 282.3 + 284.2 | 511.7 | $0.63 \pm 0.15$ | 0.08 (M1) | $3.3 \pm 0.5$ |
| 6 | 282.3 + 284.2 | 571.0 | $3.7 \pm 0.5$ | 0.053 (M1) | $65 \pm 10$ [†] |
| 7 | 284.2 | 703.3 | $2.5 \pm 0.3$ | 0.011 (E2) | $130 \pm 20$ |
| 8 | 349.4 | 571.0 | $0.87 \pm 0.25$ | 0.053 (M1) | $55 \pm 15$ |
| 9 | 349.4 | 703.3 | $0.67 \pm 0.15$ | 0.011 (E2) | $170 \pm 80$ |
| 10 | 580.0 | 571.0 | $0.27 \pm 0.11$ | 0.053 (M1) | $10 \pm 4$ |
| 11 | 910.8 | 703.3 | $0.11 \pm 0.04$ | 0.011 (E2) | $150 \pm 40$ |
| 12 | 260.5 | 703.3 | $-0.5 \pm 0.3$ | 0.011 (E2) | $<30$ |
| 13 | 260.5 | 987.8 | $-0.2 \pm 0.2$ | 0.006 (E2) | $<20$ |
| 14 | 260.5 | 1043.7 | $-0.3 \pm 0.2$ | 0.015 (M1) | $<5$ |
| 15 | 282.3 | 349.4 | $0.0 \pm 0.3$ | 0.18 (M1) | $<2$ |
| 16 | 282.3 | 1043.7 | $0.0 \pm 0.16$ | 0.015 (M1) | $<20$ |
| 17 | 284.2 | 349.4 | $0.0 \pm 0.3$ | 0.18 (M1) | $<0.5$ |
| 18 | 284.2 | 910.8 | $0.0 \pm 0.1$ | 0.006 (E2) | $<10$ |
| 19 | 284.2 | 1043.7 | $0.0 \pm 0.16$ | 0.015 (M1) | $<5$ |
| 20 | 284.2 | 1766.4 | $-0.17 \pm 0.2$ | 0.28[††] (M1) | $<20$ |
| 21 | 349.4 | 1043.7 | $-0.2 \pm 0.1$ | 0.18[††] (M1) | $<20$ |
| 22 | 571.0 | 703.3 | $0.0 \pm 0.03$ | 0.011 (E2) | $<10$ |
| 23 | 1766.4 | 703.3 | $-0.1 \pm 0.15$ | 0.011 (E2) | $<15$ |

[†] The value was calculated by assuming that the 260.5 and 282.3 keV transitions alone were responsible for the coincidence effect.

[††] $\varkappa_1$ (not $\varkappa_2$).

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[†††] The following relation holds between the K conversion probability $\varkappa$, the K conversion coefficient $\alpha_K$ and the total conversion coefficient $\alpha$: $\varkappa = \alpha_K/(1+\alpha)$.

neglected, which may introduce an error in $C_g$. This error, however, is estimated to be less than 10 %.

The solid angle $\omega_2$ was measured with a $Bi^{207}$ source (of the same shape as the $Bi^{205}$ source) and was found to be 2.5 % with a resolution of 2.8 %.

The chance coincidence rates $C_{ch}$ were measured directly by introducing into one of the channels a time delay long compared to the resolving time of the coincidence circuit. These values were in agreement with those calculated from formula (2).

The genuine coincidences $C_g$ given in table IV column 4 were obtained by subtracting the chance coincidences $C_{ch}$ from the total coincidences $C_{tot}$. The calculated errors in $C_g$ are due to the statistical errors in $C_{ch}$ and $C_{tot}$ (systematic errors were not included).

From the values of $C_g$ it is possible to decide whether two lines are in coincidence or not. Furthermore it is possible to determine the product $\delta_2\varkappa_2$. $C_g$ and $N_1$ are measured; $\varepsilon_2 \approx 1$ for electrons of energies larger than about 100 keV, (for lower energies the electron pulses could not be well resolved from the dark current pulses); $\omega_2 = 2.5$ % and $\varepsilon_c \approx 1$. Thus in cases where $\alpha_K$ (and therefore $\varkappa_2$) is known, $\delta_2$ can be calculated from formula (1) (and vice versa). Table IV column 6 includes the values of $\delta_2$ calculated by use of $\alpha_K$ values obtained from the results listed in table III. The errors in $\delta_2$ are statistical. The $\delta_2$ values will be discussed in section 3.3.

A rough summary of the results of the $e^-$—$e^-$ coincidence measurements is included in table VI. A plus sign indicated that two lines have been found to be in coincidence. The ratios $C_g/C_{ch}$ had in these cases values between 1 and 4. A minus sign in table VI signifies that no coincidence effect between the two lines in question was observed. The same table also includes the results of the electron-gamma coincidence experiments described below.

### 2.7. ELECTRON-GAMMA COINCIDENCE MEASUREMENTS

When a coincidence effect between a high energy and a low energy transition is to be investigated the $e^-$—$\gamma$ coincidence method possesses an advantage over the $e^-$—$e^-$ method, as the conversion coefficient decreases rather rapidly with energy (for a certain multipolarity). In addition, the transmission in the gamma channel is high. On the other hand, the resolution of the gamma channel is poorer. For low energy gamma rays the $e^-$—$\gamma$ method is not convenient because of the interference of the Compton spectrum of all the higher energy gamma rays.

In this work the $e^- - \gamma$ method was applied especially to the investigation of coincidence effects between the 1614.6, 1766.4 and 1863.6 keV gamma rays and conversion lines of energy lower than 700 keV.

The $e^- - \gamma$ coincidence measurements were performed with the source described in section 2.6. The double long lens $e^- - e^-$ spectrometer [9]) was adapted for the purpose, as can be seen in fig. 7. The spectrometer was opened on one side and a separate gamma-detector unit introduced. This consists of a NaI (Tl) scintillation crystal attached to a lucite light guide and a photomultiplier. The efficiency of the light guide has been measured to be nearly 70 percent and the pulse height resolution is practically the same as is obtained without any light guide at all. The distance between the source and the crystal could be varied from 0 up to about 5 cm. Different kinds and sizes of crystals could be used. Suitable absorbers were inserted between the source and the crystal. Lead plates of thickness 3 or 7 mm were used in order to shield off the low energy lines. Thus, stronger sources could be used without overloading the electronics.

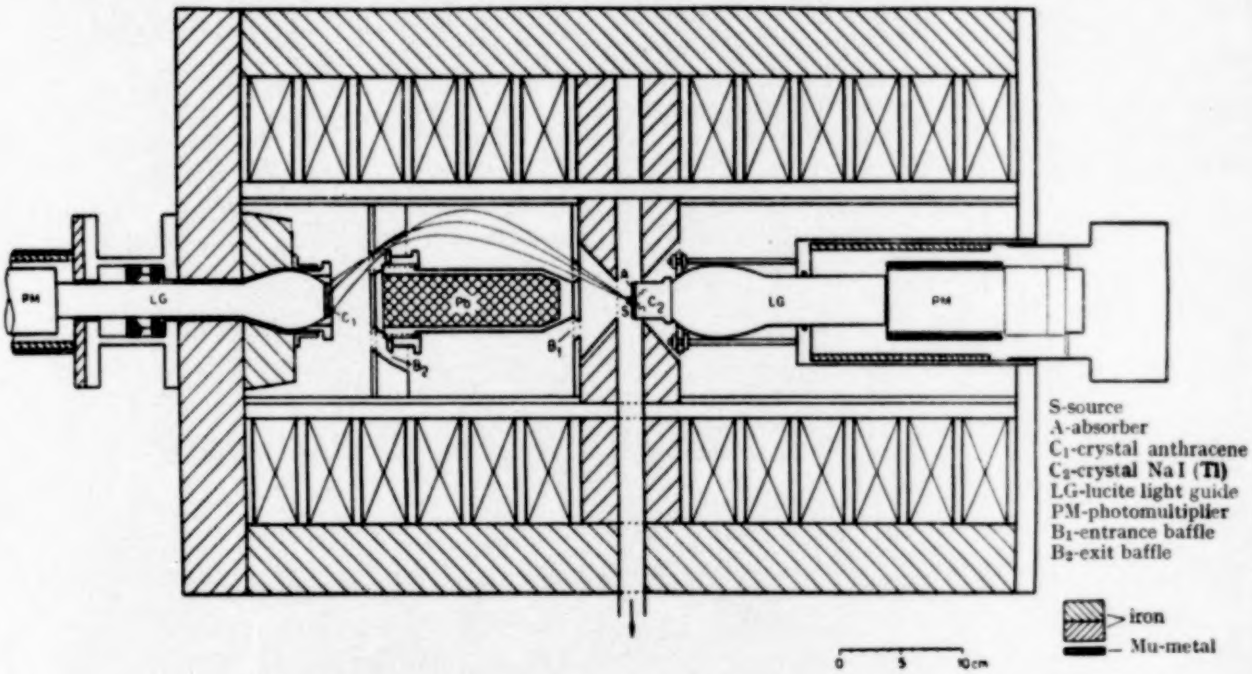

PM
LG
$C_1$
Pb
$B_1$
$B_2$
A
S
$C_2$
LG
PM
S-source
A-absorber
$C_1$-crystal anthracene
$C_2$-crystal NaI (Tl)
LG-lucite light guide
PM-photomultiplier
$B_1$-entrance baffle
$B_2$-exit baffle
iron
Mu-metal
0
5
10cm

Fig. 7. The electron-gamma coincidence arrangement described in section 2.7.

The shielding of the photomultiplier tube was very efficient and no influence from the magnetic field in the opposite half of the instrument could be observed.

The procedure of measurement was the following: The magnetic field was adjusted so that the peak of a conversion line was focussed in the electron channel. Then, keeping the magnetic field constant, a differential or an integral spectrum in coincidence with the conversion line was taken by changing the discrimination level in the gamma channel.

TABLE V

Electron-gamma coincidences in the decay of $Bi^{205}$

| Pair symbol | Energy (in keV) of transition observed in | | Assumed value of $\varkappa_{e^-}$ | Calculated value of $\delta_{e^-}$ (%) |
|---|---|---|---|---|
| | $\gamma$-channel | $e^-$-channel | | |
| a | 1766.4 | 260.5 | 0.64 (M2) | $< 1$ |
| b | 1766.4 | 282.3 | 0.28 (M1) | $< 1$ |
| c | 1766.4 | 284.2 | 0.28 (M1) | $< 0.05$ |
| d | 1766.4 | 349.4 | 0.18 (M1) | $< 0.5$ |
| e | 1766.4 | 571.0 | 0.053 (M1) | $< 1$ |
| f | 1766.4 | 703.3 | 0.011 (E2) | $<10$ |
| g | 1766.4 | 987.8 | 0.006 (E2) | $<10$ |
| h | 1766.4 | 1043.7 | 0.012 (M1) | $< 0.5$ |
| i | 1614.6 | 260.5 | 0.64 (M2) | 20 |
| j | 1863.3 | 703.3 | 0.011 (E2) | 80 |
| k | 987.8+1043.7 | 703.3 | 0.011 (E2) | $<10$ |

The upper limits of $\delta_{e^-}$ for negative coincidence effects with any (A) of the 1777.4, 1863.3 and 1906.5 keV gamma rays are obtained from the limits given above for the corresponding coincidences with the 1766.4 keV gamma ray by multiplying by the gamma intensity ratio $\gamma 1766.4/\gamma A$.

The number of genuine coincidences per unit time is

$$C_g = N_\gamma \delta_{e^-} \varkappa_{e^-} \omega_{e^-} \varepsilon_{e^-} \varepsilon_c$$

where the symbols have the corresponding meanings to those in formula (1) of section 2.6. $\varepsilon_{e^-} \approx 1$ and $\varepsilon_c \approx 1$.

Table V includes some estimated values of $\delta_{e^-}$ for some high energy gamma rays, (see also section 3.3). A rough summary of the results of the $e^-$—$\gamma$ coincidence measurements is included in table VI.

A positive coincidence effect was found between the K 703.3 conversion line focused in the electron channel, and a gamma ray of 0.5 MeV. The effect was much bigger than was expected for the coincidence pair 703.3—511.7. The 0.5 MeV gamma ray in question is believed to be the annihilation radiation, thus showing that the positrons emitted from $Bi^{205}$ (see section 2.3) are in coincidence with the 703.3 keV transition.

TABLE VI

Electron-electron and electron-gamma coincidences in the decay of $Bi^{205}$

| Transition energy (keV) | 260.5 | 282.3 | 284.2 | 349.4 | 511.7 | 571.0 | 703.3 | 987.8 | 1043.7 |
|---|---|---|---|---|---|---|---|---|---|
| 260.5 | | + | | + | | + | − | − | − |
| 282.3 | + | | + | − | + | + | | | − |
| 284.2 | | + | | − | | | + ⊕ | | − |
| 349.4 | + | − | − | | | + | + | | − |
| 580.0 | | | | − | | + ? | | | |
| 703.3 | − | | + ⊕ | + | | − | | ⊖ | ⊖ |
| 911.7 | | | − | | | | + ⊕ | | |
| 1614.6 | ⊕ | | | | | | | ⊖ | |
| 1766.4 | − ⊖ | ⊖ | − ⊖ | − | | ⊖ | − ⊖ | ⊖ | ⊖ |
| 1777.4 | − ⊖ | ⊖ | ⊖ | − | | ⊖ | − ⊖ | | ⊖ |
| 1863.3 | ⊖ | ⊖ | ⊖ | | | | ⊕ | | |
| 1906.5 | ⊖ | ⊖ | ⊖ | | | | | | |

+ = coincidence effect observed in $e^- - e^-$ coincidence experiments.
− = no coincidence effect observed in $e^- - e^-$ coincidence experiments.
⊕ = coincidence effect observed in $e^- - \gamma$ coincidence experiments.
⊖ = no coincidence effect observed in $e^- - \gamma$ coincidence experiments.

## 3. Construction of the Decay Scheme

### 3.1. TOTAL DECAY ENERGY AND POSITON FEEDING

For construction of the decay scheme it is valuable to know the total decay energy of $Bi^{205}$. This can be deduced as will be shown below, from the available results of measurements.

As described in section 2.4, a gamma ray of 2.6 MeV was founds in the scintillation spectrum. The end point of the Fermi plot of the positon spectrum (fig. 4b) indicates a maximum energy of 925 keV for positon emission. It is clear therefore that the positons cannot feed any level *below* $2.60-0.925-1.022 \approx 0.65$ MeV. The scintillation spectrum taken with a source inside a hollow crystal (section 2.4) shows that there is no sum-up to an energy higher than 2.60 MeV. It is reasonable to assume therefore that the total decay energy cannot be much higher than 2.6 MeV. Thus the positons found cannot feed any level above about 1 MeV.

It is clear from the energy level scheme (fig. 8) that only two levels can be fed by the positons of 925 keV maximum energy, namely those at 703.3 and 987.6 keV.

Coincidence measurements (section 2.7) showed a coincidence effect between the 703.3 keV conversion line and annihilation radiation. Thus we get for $Bi^{205}$ a total decay energy of

$$(0.703+(0.925\pm 30)+1.022)\ \text{MeV},$$

that is

$$E_d = (2.65\pm 0.03)\ \text{MeV},$$

and for the mass of $Bi^{205}$ (taking $M_{Pb^{205}} = 205.03800$ [16]))

$$M_{Bi^{205}} = (205.04085\pm 0.00003)\ \text{m.u.}$$

### 3.2. ENERGIES OF LEVELS IN $Pb^{205}$

Because of the high accuracy of the spectrometers it is justified to start the construction of the level scheme by considering the gamma ray energies. For this purpose the differences of all possible pairs of the gamma ray energies given in table I were calculated. In table VII below are included only those cases where the sum $E_{\gamma 1}+E_{\gamma 2}$ of two gamma ray energies differ from the energy $E_{\gamma 3}$ of a third gamma ray by less than 1 : 1000. This criterium is justified by the accuracy of the gamma ray energies. In spite of this, about one third of the sums listed in table VII seem to be accidental.

The construction of a level scheme by using sum relations like 1—17 is considerably simplified if one excited state, for example the first excited state, is known, as was the case with $Pb^{205}$. Both Harvey's results [5]) and $\alpha$ decay measurements [3]) indicate that the first excited state in $Pb^{205}$ should have an energy around 270 keV. There are three weak gamma rays (260.5, 282.3 and 284.2 keV) in this energy region, however, and for this reason another starting point has to be used. Interest should be focused on the 703.3, 987.8, 1043.7, and 1766.4 keV $\gamma$-rays which are the strongest ones. None of these gamma rays is in coincidence with any of the other three, which suggests that the gamma rays are due to transitions directly to the ground state from levels of the same energies. Using the sums 3, 2, 11, 12, and 13 and the arguments mentioned above, levels of energies 703.3, 987.6, 1043.7, 1614.6 and 1766.4 keV are obtained, from which gamma rays of energies 284.2, 571.0, 626.5, 703.3, 910.8, 987.8, 1043.7, 1614.6, and 1766.4 keV are emitted. This suggested level sequence is also strongly supported by the coincidence measurements (see tables IV, V and VI) and should be considered as a very good start for constructing the rest of the level scheme.

TABLE VII

Sum relations in $Pb^{205}$

| Sum no. | $E_{\gamma1}$ | $E_{\gamma2}$ | $E_{\gamma1}+E_{\gamma2}$ | $E_{\gamma3}$ | $\frac{E_{\gamma1}+E_{\gamma2}-E_{\gamma3}}{E_{\gamma3}}$ |
|---|---|---|---|---|---|
| 1 | 115.2 | 511.7 | 626.9 | 626.5 | 7 : $10^4$ |
| 2 | 284.2 | 626.5 | 910.7 | 910.8 | 1 : $10^4$ |
| 3 | 284.2 | 703.3 | 987.5 | 987.8 | 3 : $10^4$ |
| 4 | 282.3 | 761.0 | 1043.3 | 1043.7 | 4 : $10^4$ |
| 5 | 493.6 | 550.0 | 1043.6 | 1043.7 | 1 : $10^4$ |
| 6 | 511.7 | 531.7 | 1043.4 | 1043.7 | 3 : $10^4$ |
| 7 | 349.4 | 987.8 | 1337.2 | 1337 | |
| 8 | 349.4 | 1002.3 | 1352.1 | 1351.5 | 5 : $10^4$ |
| 9 | 745.0 | 758.3 | 1593.3 | 1502.5 | 5 : $10^4$ |
| 10 | 550.0 | 1002.7 | 1552.7 | 1552.9 | 5 : $10^4$ |
| 11 | 571.0 | 1043.7 | 1614.7 | 1614.6 | 0.7 : $10^4$ |
| 12 | 626.5 | 987.8 | 1614.3 | 1614.6 | 2 : $10^4$ |
| 13 | 703.3 | 910.8 | 1614.1 | 1614.6 | 3 : $10^4$ |
| 14 | 511.7 | 1351.5 | 1863.2 | 1863.3 | 0.6 : $10^4$ |
| 15 | 571.0 | 1337 | 1908.0 | 1906.5 | 8 : $10^4$ |
| 16 | 703.3 | 1863.3 | 2566.6 | 2600† | |
| 17 | 1552.3 | 1014.2 | 2566.5 | 2600† | |

† Found in scintillation spectrum only.

The sum relations, which have not been included so far will be discussed below in a convenient order together with the corresponding coincidence information, (cf. the level scheme of fig. 8).

*Energy sum No.* 4 (282.3+761.0): The 282.3 and 761.0 keV transitions are too weak for coincidence measurements. However, their energies strongly suggest that they are emitted in cascade from the 1043.7 keV level. It is preferable to assume that the 282.3 keV gamma ray is emitted after the 761.0 keV gamma ray, since this indicates a low lying level of the energy 282.3 keV. This is probably identical with the level observed by the Berkeley group [4]) and by Harvey [5]), because as was shown above the 284.2 keV gamma ray is emitted from the 987.6 keV level, and the 260.5 keV gamma ray will be shown to be emitted from a high lying level.

*Energy sums No.* 16 *and* 17 (703.3+1863.3 *and* 1552.3+1014.2): Sum 16 suggests that the 1863.3 keV transition should be emitted from a level of energy 2566.5 keV to the 703.3 keV level. This is in agreement with the coincidence measurements, which showed that the 1863.3 and 703.3 keV transitions are in coincidence (cf. table V, pair *j*). Sum 17 suggests that also the 1014.2 and 1552.0 keV gamma rays are emitted in cascade from the same level. It is not possible to say anything about the succession of the 1014.2 and the 1552.0 keV gamma rays, and the intermediate level may therefore have the energy 1014.2 or 1552.3 keV. Very probably the 2.60 MeV gamma ray, which was found in the scintillation measurements, is emitted from the 2566.5 keV level.

*Energy sum No.* 14 (511.7+1351.5)*:* This sum also suggests that the 511.7 and 1351.5 keV gamma rays are emitted in cascade from the 2566.5 keV level. One has to assume that the 511.7 keV gamma ray preceeds the 1351.5 keV gamma ray in order to explain the coincidence of the 511.7 keV gamma ray with either the 282.3 or the 284.2 keV gamma rays (cf. table IV, pair 5). The new level thus introduced has the energy 2054.8 keV.

*Energy sum No.* 8 (349.4+1002.3)*:* The energy sum suggests that the 349.4 and 1002.7 keV gamma rays are emitted in cascade from the 2054.8 to the 703.3 keV level. This is in agreement with the coincidence measurements where the 349.4 and 703.3 keV gamma rays were found to be in coincidence (cf. table IV, pair 9). One has to assume that the 349.4 keV gamma ray preceeds the 1002.7 keV gamma ray in order to explain the observed coincidence effect between the 571.0 and 349.4 keV gamma rays (cf. table IV, pair 8). This introduces a new level at the energy 1705.4 keV.

*Energy sum No.* 5 (493.6+550.0)*:* The 493.6 and 550.0 keV gamma rays are probably emitted in cascade from the 1043.7 keV level, which, however, could not be checked by coincidence measurements for intensity reasons. It is not possible to say anything about the succession, but the intermediate level may have the energy 550.0 or 493.6 keV.

*Energy sum No.* 9 (745.0+758.3) *:* The 745.0 and 758.3 keV gamma rays are too weak for coincidence measurements. Since the 1502.5 keV gamma ray is not found in any other combination it seems possible that this gamma ray is emitted from a level of the same energy. This level would also emit the 745.0 and 758.3 keV gamma rays in cascade, but the succession is not possible to determine. The intermediate level may have the energy 745.0 or 758.3 keV.

*Energy sum No.* 1 (115.2+511.7)*:* This sum relation is accidental because of the positions of the 511.7 and 626.5 keV gamma rays in the level scheme.

*Energy sum No.* 6 (511.7+531.7)*:* The 511.7 and 531.7 keV gamma rays could be emitted in cascade from the 1043.7 keV level. This would be in disagreement with the positions of the 511.7 and the 1043.7 keV transitions in the level scheme. The energy agreement is therefore probably accidental.

*Energy sum No.* 7 (349.4+987.8)*:* The 349.4 keV gamma ray is in coincidence with the 571.0 keV gamma ray (cf. table IV, pair 8), from which it must be concluded that the sum-up is accidental.

*Energy sum No.* 10 (550.0+1002.7)*:* The energy agreement is probably accidental because the 1002.7 keV gamma ray fits into another suggested level sequence (see sums 8, 14 and 16) which accounts for more gamma rays and is therefore preferable.

*Energy sum No.* 15 (571.0+1337)*:* This sum relation may be accidental because the 571.0 keV gamma ray probably goes to the 1043.7 keV level, and this would mean that the 1906.5 keV gamma ray would be emitted from a level of the energy 2950 keV. This is larger than the disintegration energy and therefore impossible.

There is no sum relation which includes the 1190.3 keV level. This gamma ray may therefore be emitted to the ground state from a level of the same energy. This may also be the case for the 1337 and 1906.5 keV gamma rays, since the sums 7 and 15 proved to be accidental.

The 260.5 keV gamma ray is the only gamma ray for which considerable coincidence information is available but which has not yet been discussed. It is in coincidence with the 282.3 or 284.4, 349,4, 571.0, 703.3 and the 1614.6 keV gamma rays. This definitely suggests that the 260.5 keV gamma ray is emitted from a high lying level. It is possible that the 260.5 keV gamma ray is emitted between a level of energy 2315.3 keV and the 2054.8 keV level. If this is the case, to be able to explain how the 260.5 keV gamma ray can be in coincidence with a) the 282.4 or 284.3, 571.0, 703.3 and 1614.6 keV gamma rays and b) the 349.4 keV gamma ray it must be assumed that there is a 90.8 keV transition between the 1705.4 and 1614.6 keV levels. Table II includes some conversion lines, which may be the L-lines of such a transition ($E_\gamma = 90.0$ keV).

The discussion of the levels in $Pb^{205}$, which was based on energy, intensity and coincidence measurements, has resulted in 18 levels below 2600 keV. Of these levels 10 (solid lines in fig. 8) are considered to be well established, while the other 8 (dotted lines in fig. 8) are more uncertain, since coincidence measurements are lacking for intensity reasons. The best established part of the level scheme of $Pb^{205}$ is presented on the left of fig. 8, while the right part includes some uncertainties. In cases where there are several ways to reach the ground state from a given level, the accepted level energies are weighted mean values. As can be seen in table 8, different transition possibilities to the ground state have energies which agree very satisfactorily.

TABLE VIII

Deduction of the level energies in $Pb^{205}$

| Sum of transition energies (keV) | Energy of transition to the ground state (keV) | Accepted level energy (keV) |
|---|---|---|
| 282.3 + 703.3 = 987.6 | 987.8 | 987.6 |
| 761.0 + 282.3 = 1043.3<br>493.6 + 550.0 = 1043.6 | 1043.7 | 1043.7 |
| 758.3 + 745.0 = 1503.3 | 1502.5 | 1503.0 |
| 1614.6 + 90.0 = 1704.6<br>1002.7 + 703.3 = 1706.0<br>717.5 + 987.8 = 1705.3 | – | 1705.4 |
| 349.4 + 1002.7 + 703.3 = 2055.4<br>1351.5 + 703.3 = 2054.8 | – | 2054.8 |
| 1863.3 + 703.3 = 2566.6<br>1014.2 + 1552.0 = 2566.2<br>2054.8 + 511.7 = 2566.5<br>2055.4 + 511.7 = 2567.1 | 2600 † | 2566.5 |

† Found in scintillation spectrum only.

For cases which are not included in table 8 the level energies are based on only one gamma ray energy. Using the adopted level energies, all possible differences of two pairs of levels were calculated. In five cases energy values were obtained which may correspond to the energies of doubtful gamma rays of table II.

These possible gamma transitions are dotted in the right part of fig. 8.

### 3.3. BRANCHING RATIOS

The decay scheme constructed in the preceding section is based on the accurate gamma energies and on the qualitative coincidence information. Its agreement with the branching ratios can now be checked (cf. tables IV and V). The calculated branching ratios are not very accurate, due to poor statistics. Furthermore, the electronics are not completely reproducible, so that systematic errors might be introduced during the long measurements. In spite of these reservations the branching ratios provide a valuable check for the proposed decay scheme.

Starting with the positive $e^-$—$e^-$ coincidence results, it is seen (cf. table IV and fig. 8) that the results of pairs 7 and 11 are in good agreement with the decay scheme, indicating that the 703.3 keV level is de-excited predominantly by the 703.3 keV transition directly to the ground state.

The $\delta_2$ value of pair 8 in table IV does not fit very well with $\delta_2$ of pair 9, as they are both too high (expected to be less than 100 together). From the relative intensities of the transitions emitted from the 1614.6 keV level it is found that the first $\delta_2$ value is about twice too high. The disagreement is probably within experimental error.

The result of pair 6 is readily understood if the 282.3 keV line is responsible for the coincidence effect.

The $\delta_2$ values of pairs 1, 2, 3, 4, and 5 are in reasonable agreement with the decay scheme. However, detailed comparisons are difficult to make due to insufficient information on the intensities of several high energy transitions de-exciting the high lying levels.

Turning to the negative coincidence effects, (see lower part of table IV), it is seen that the results of pairs 12, 13, and 21 are not necessarily in disagreement with the decay scheme, because the lower limits for $\delta_2$, set by these measurements, are rather high and thus not significant.

The results of pairs 14 and 15 are also not in disagreement with the decay scheme. The $\delta_2$ values for these pairs (cf. pairs 3 and 6) are expected to be small, and the coincidence effect is probably just on the limit of observability.

The $e^-$—$\gamma$ results apply mainly to the strong 1766.4 keV gamma

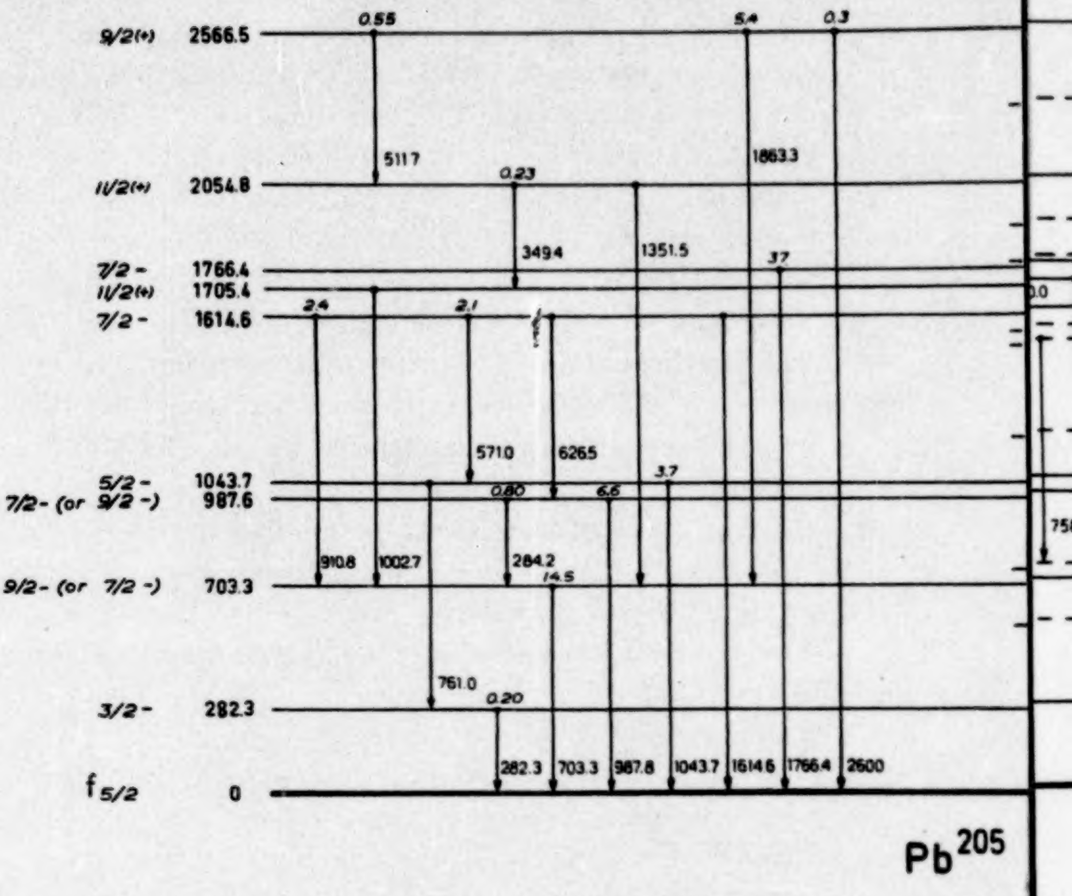
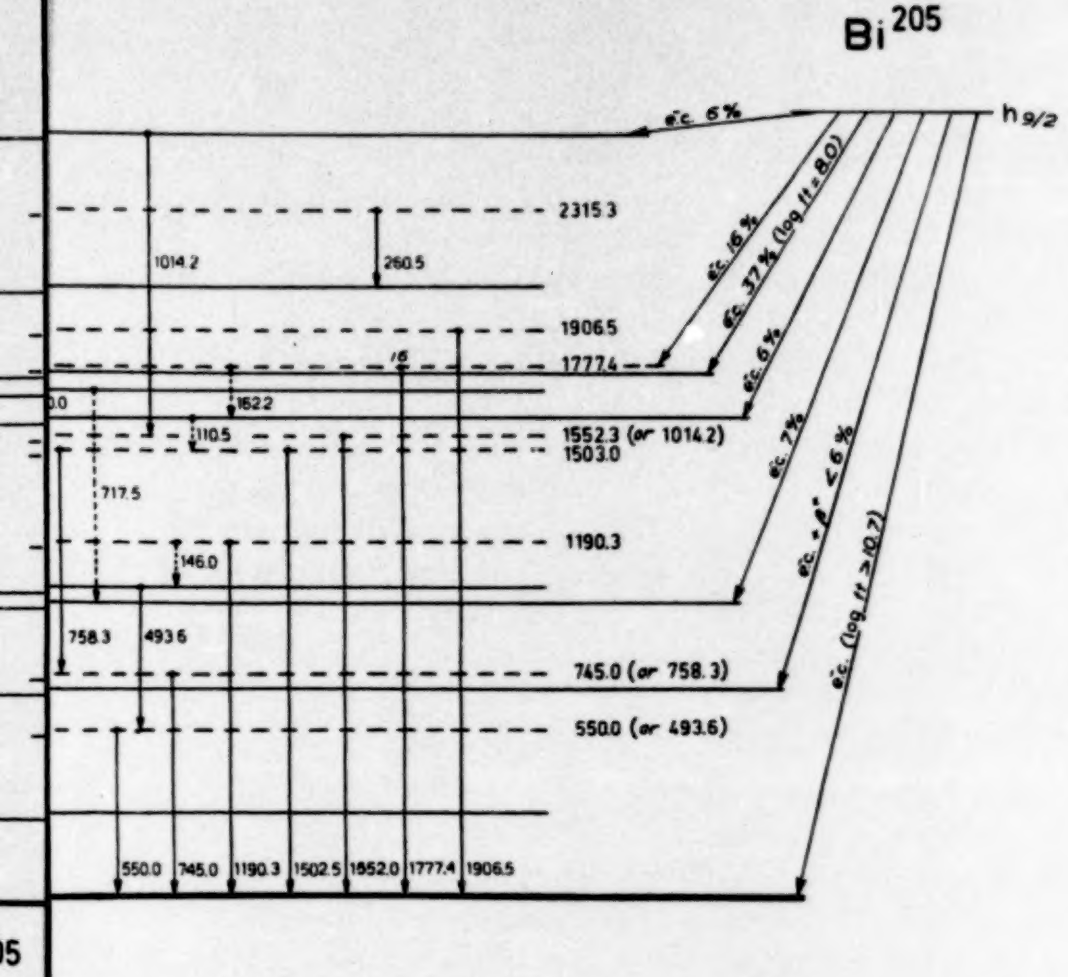

9/2(+) 2566.5
11/2(+) 2054.8
7/2 - 1766.4
11/2(+) 1705.4
7/2 - 1614.6
5/2 - 1043.7
7/2- (or 9/2 -) 987.6
9/2- (or 7/2 -) 703.3
3/2 - 282.3
f5/2 0
0.55
5.4
0.3
511.7
1863.3
0.23
349.4
1351.5
3.7
2.4
2.1
571.0
626.5
3.7
0.80
6.6
910.8
1002.7
284.2
14.5
761.0
0.20
282.3
703.3
987.8
1043.7
1614.6
1766.4
2600
Pb205

Fig. 8. Decay scheme of Bi$^{205}$. The left part of the level scheme includes only levels nd
dotted lines to the right indicate levels and transitions which are considered as not uit
percent per isi


$Bi^{205}$
$h_{9/2}$
e.c. 6%
e.c. 16%
e.c. 37% (log ft ≈ 8.0)
e.c. 6%
e.c. 7%
e.c. + β⁺ < 6%
e.c. (log ft ≥ 10.7)
2315.3
1906.5
1777.4
1552.3 (or 1014.2)
1503.0
1190.3
745.0 (or 758.3)
550.0 (or 493.6)
1014.2
260.5
16
162.2
110.5
717.5
146.0
758.3
493.6
550.0
745.0
1190.3
1502.5
1552.0
1777.4
1906.5
05

vels nd transitions which are considered as definitely established (heavy lines). The
not uite certain. At the head of some of the transitions are given the intensities in
per isintegration.

ray (cf. section 2.7 and table V). The $\delta_e$-value of pair $i$ is somewhat high (cf. pair 3 in table IV and the relative gamma intensities of the 571.0 and 1614.6 keV transitions), but still well within the (large) error of this measurement.

### 3.4. ELECTRON CAPTURE FEEDING

After constructing the energy level scheme, it is possible to estimate the relative intensities of electron capture feedings to some of the levels. The results, in percent per disintegration, are shown in fig. 8. The e$^-$.c. feeding to a level was deduced by subtracting the total intensity of the transitions feeding this level from the total intensity of the transitions de-exciting it. The log *ft* values were calculated with the aid of Moszkowski's monogram [17]) and assuming a total decay energy of 2.65 MeV.

The lower limit set for the log *ft* value for electron capture to the ground state was deduced as follows: 1) An upper limit for the intensity of a possible high energy component in the positon spectrum was estimated, taking into account the statistic errors in the measurements. 2) The total feeding (e$^-$.c.$+\beta^+$) to the 703.3 keV level was estimated to be less than 6 %. 3) The energy dependence of the ratio $\beta^+$/e$^-$.c. was assumed to be the same as that calculated for allowed transitions.

### 3.5. ASSIGNMENT OF SPINS AND PARITIES

The assignment of spins and parities on the basis of the experimental material alone is very difficult, so that the following discussion is speculative in character (in the following paper Pryce attempts to correlate the experimental decay scheme with the theoretically predicted levels).

*The ground state:* The spin and parity of the long-lived ground state of $Pb^{205}$ has not been measured directly but an assignment 5/2— seems most probable for the following reasons:

1) Studies of the isomeric transitions in odd lead isotopes (Stockendal *et al.* [11])) have shown that the ground states of $Pb^{201}$ and $Pb^{203}$ most probably have the spins 5/2—, and the ground state of $Pb^{207}$ is 1/2— (odd numbers of holes in the $f_{5/2}$ and $p_{1/2}$ shells). It therefore seems very plausible to assume that the ground state of $Pb^{205}$ has the spin 5/2— due to the configuration $f_{5/2}^{-1}\,p_{1/2}^{-2}$.

2) The ground state of $Bi^{205}$ is $h_{9/2}$ according to the shell theory (cf. the spin 9/2— for $Bi^{209}$). The fact that positons were found in the decay of $Bi^{205}$, which, however, do not go to the ground state

with any observable intensity, suggests a spin difference of at least 2 units i.e. a spin $\geqq 5/2$ for the ground state of Pb$^{205}$.

3) It might be possible to deduce the spin of the ground state of Pb$^{205}$ from the *ft*-value of the electron capture feeding to Tl$^{205}$. Quite recently Pb$^{205}$ was shown to decay by L capture to Tl$^{205}$ with a half life of $5 \times 10^7$ years [18]). No gamma rays were detected with an energy above 12 keV (L X-rays), and therefore we believe that the L capture goes to the 1/2+ ground state of Tl and not to the 205 keV 3/2+ first excited state, as might have been expected from the energy cycle (the large value 380 keV for the energy difference between the ground states of Pb$^{205}$ and Tl$^{205}$, obtained from the *Q*-values of the reactions Tl$^{206}$ $\xrightarrow{\beta^-}$ Pb$^{206}$, Tl$^{205}$ (n, $\gamma$) Tl$^{206}$ and Pb$^{206}$ (d, t) Pb$^{205}$ is probably due to errors in the *Q*-values of the latter two reactions). [19]) No calculations of log *ft* for L capture are known to us, but in the case of an allowed L capture in Bi$^{207}$ a half life of $10^2$ years was found [20]), so that a half life of $5 \times 10^7$ years for a second forbidden transition (5/2— to 1/2+) might not be unreasonable.

The assignment below of spin and parity values is based on the knowledge of the transitions and on the assumption that the spins of the ground states of Bi$^{205}$ and Pb$^{205}$ are $h_{9/2}$ and $f_{5/2}$ respectively. Below are given the arguments for the level assignments of Pb$^{205}$ given in fig. 8.

*The* 282.3 *keV level* may have the spin 3/2—, 5/2— or 7/2—, because the transition between this level and the ground state is predominantly of multipolarity M1. The most probable spin for the 282.3 keV level is 3/2— since the spins 5/2— and 7/2— must be excluded because of the absence of strong feeding from the $h_{9/2}$ level in Bi$^{205}$ and the weak feeding from higher levels in Pb$^{205}$.

*The* 703.3 *keV level* certainly has the spin 9/2— or 7/2— because the transition to the 5/2— ground state is mainly of multipolarity E2 (though an admixture of about 20 % M1 cannot be excluded). The spins 1/2—, 3/2— and 5/2— are impossible because of the capture feeding to the 703.3 keV level. For reasons given in the following, the 9/2— assignment is preferable.

*The* 987.6 *keV level* has the spin 7/2— or 9/2— for the same reasons as this assignment was chosen for the 703.3 keV level. As will be shown in the following, the assignment 7/2— is preferable.

*The* 1043.7 *keV level* was assigned the spin 5/2—, for the following reasons: The 1043.7 keV transition to the 5/2— ground state is of multipolarity M1. The spin of the 1043.7 keV level might therefore be 3/2—, 5/2— or 7/2—. The spin 7/2— is excluded because of the absence of a transition to the 9/2— 703.3 keV level. The spin 3/2— is impossible since the 1614.6 keV level certainly has the spin 7/2— or 9/2— and the transition between the 1614.6 and 1043.7 keV levels is of multipolarity M1.

*The* 1614.6 *keV level* is connected with the 5/2— ground state of $Pb^{205}$ through the 1614.6 keV transition and with the 9/2— 703.3 keV and 7/2— 987.6 keV levels through transitions (910.8 and 626.5 keV) which are predominantly of multipolarity E2. In addition, there is capture feeding from the $h_{9/2}$ ground state of $Bi^{205}$. The spin of the 1614.6 keV level is therefore certainly either 9/2— or 7/2—. 7/2— is the most probable assignment because of the 571.0 keV M1 transition to the 5/2— 1043.7 keV level.

*The* 1705.4 *keV level* should have a rather high spin because the only transition observed from this level is the weak 1002.7 keV transition to the 703.3 keV level. The spin of the 1705.4 keV level might be 11/2. It is then plausible to assume that the 703.3 keV level has the spin 9/2— and that the 987.8 keV level has the spin 7/2— (both up till now either 7/2— or 9/2—), which might explain why the transition to the 987.6 keV level was not observed.

*The* 1766.4 *keV level* is strongly fed by electron capture and should therefore have the spin 11/2—, 9/2— or 7/2—. The strong transition to the 5/2— ground state is mainly of multipolarity E2, though an admixture of M1 cannot be excluded. For this reason the spin 11/2— is impossible. However, it cannot be decided from the experimental material whether the 1766.4 keV level has the spin 9/2— or 7/2—. As will be shown later the 7/2— assignment is preferable.

*The* 2054.8 *keV level* should also have a fairly high spin because there are only two transitions from this level, viz. the 349.4 keV M1 transition going to the 1705.4 (11/2) keV level and the 1351.5 keV transition going to the 703.3 keV level. It should be noticed that also in this case (cf. the 1705.4 keV level) a transition to the 987.6 keV level is not observed. For these reasons a spin of 11/2 seems most plausible for the 2054.8 keV level.

*The* 2566.5 *keV level* is relatively strongly fed by electron capture in spite of the fact that the decay energy is so small that most of the capture probably takes place by L capture. This might indicate an allowed transition and suggests the spin 11/2—, 9/2— or 7/2—. 9/2— seems more probable than the other possibilities.

However, the probable multipolarity assignment of the 1863.3 keV transition as E1 suggests that the 2566.5 keV level has even parity, and in that case the 1705.4 and 2054.8 keV levels should also have even parity (because of the M1 transitions of 511.7 and 349.4 keV).

We have not tried to make any assignments for the uncertain levels, except for the level at 550.0 (or 493.6) keV, which is of interest for the shell model because of its low energy. There is no capture feeding to this level. The gamma ray feeding this level is emitted from the 5/2— 1043.7 keV level and the gamma ray emitted from the 1043.7 keV level goes to the 5/2—ground state. For these reasons the spins 1/2— or 3/2— seem possible.

The proposed decay scheme contains all 34 gamma transitions of table I, except for the weak 115.2, 235.9, 383.2, 580.0, 688.9 and 1337 keV transitions. Some of these transitions appeared in numerous sum-up relations, but as they could not be checked by coincidence experiments their positions were considered too doubtful to be included in the decay scheme.

## 4. Discussion

1. As mentioned in section 2.5, no isomeric transition was found in $\mathrm{Pb}^{205}$. This fact is understandable since the metastability of the $i_{13/2}$ level (expected at 980 keV) is destroyed by the 9/2— (7/2—) level at 703.3 and possibly also by the 7/2— (9/2—) level at 987.6 keV. The reappearance of the M 4 transition in $\mathrm{Pb}^{203}$, $\mathrm{Pb}^{201}$, $\mathrm{Pb}^{199}$, and $\mathrm{Pb}^{197}$ indicates that the $i_{13/2}$ level in these isotopes has lower energy than the 9/2— (and 7/2—) levels. Indeed, we found experimentally that the energy difference $i_{13/2} \rightarrow f_{5/2}$ in odd lead isotopes decreases rather rapidly with increasing distance from the double magic ${}^{126}_{82}\mathrm{Pb}^{208}$, while the energies of the 9/2—and 7/2—states presumably increase or, at least do not decrease so rapidly, relative to the 5/2— ground states. In addition to the destruction of the metastability of the $i_{13/2}$ level in $\mathrm{Pb}^{205}$, there is also very little feeding to this level, otherwise we should be able to observe the M2 transition $i_{13/2} \rightarrow 9/2-$ and/or the E3 transition $i_{13/2} \rightarrow 7/2-$ which have easily identifiable conversion properties.

This lack of feeding also differentiates between $\mathrm{Pb}^{205}$ and the other odd Pb isotopes. It can be explained as follows: In $\mathrm{Pb}^{207}$

the electron capture transition from $Bi^{207}$ to the $i_{13/2}$ level is first forbidden, while all other e⁻.c. transitions are either more highly forbidden (to $p_{1/2}$, $p_{3/2}$, $f_{5/2}$) or the energy available for them is very small (to $f_{7/2}$). Thus, although the log *ft* value for the electron capture transition to the $i_{13/2}$ level is high (9.6) [20]) it still accounts for 87 % of all the e⁻.c. feeding. In the other odd lead isotopes the log *ft* value for the electron capture transitions $h_{9/2} \rightarrow i_{13/2}$ might also be about 9.6, but as these more neutron-deficient isotopes have much shorter half-lives, this would mean that a much smaller branching will occur to the $i_{13/2}$ levels. In $Pb^{203}$, $Pb^{201}$, $Pb^{199}$ and $Pb^{197}$ transitions from higher levels (7/2—, 9/2—, 11/2—) would compensate this effect to some extent. In $Pb^{205}$, however, such a compensation presumably does not occur, probably because the low-lying 703.3 keV level (and perhaps also the 987.6 keV level) competes strongly with the $i_{13/2}$ level for gamma feeding from higher levels.

2. It is of interest to compare our level scheme with Harvey's results [5]). The levels observed by him should be "one hole levels" since the $Pb^{206}$ (d, t)$Pb^{205}$ reaction is essentially a neutron pick-up reaction. The first excited state observed by Harvey has the energy 0.27 MeV and was assigned the spin $p_{3/2}$ (α decay from $Po^{209}$ indicates a first excited state of energy 0.26 MeV [3])). Though we have no experimental proof of the succession of the 761.0 and 282.3 keV transitions, it seems very probable that the 282.3 keV transition is emitted from a level of the same energy and that Harvey and we are observing the same level. His spin assignment was based only on shell-model expectations, but our experimental results show that this level does indeed have the spin 3/2—.

The next level observed by Harvey had the energy 0.84 MeV and was assigned the spin $i_{13/2}$, again from the shell model. This energy is in fair agreement with the value 0.98 MeV obtained by interpolation from an energy versus neutron number curve for the odd M4 isomers in Pb [11]).

Harvey also observes a well defined triton group corresponding to a level at 1.79 MeV to which he assigned the spin $f_{7/2}$ (shell model expectation). It seems very plausible (Pryce) to assume that our 7/2— 1766.4 level is the same as that observed by Harvey. The transition from this $f_{7/2}$ state to the $f_{5/2}$ ground state should therefore be very dominant due to its high energy and because of the fact that it is a "one hole transition". Transitions to the 9/2— (or

7/2−) 703.3 keV level and to the 7/2− (or 9/2−) 987.6 keV level should be weaker, firstly because of their lower energies and secondly because these levels are very probably not "singly hole levels". Therefore transitions from the $f_{7/2}$ level may involve considerable nuclear rearrangements. However, we cannot completely exclude the possibility of a transition between the 1766.4 and 703.3 keV levels because such a transition would have the energy 1063.1 and the corresponding gamma ray would have almost exactly the same energy as the isomeric M4 gamma ray (1063.9 keV) in $Pb^{207}$ which is present in our source.

To a zero order approximation, the single particle levels of $Pb^{205}$ should have similar energies to the corresponding $Pb^{207}$ levels [20]), except for the $p_{1/2}$ level, which should be above the $f_{5/2}$ level. Thus, we should expect the sequence $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $i_{13/2}$ and $f_{7/2}$ levels to have the energies 0, 0.331, 0.569, 1.064 and 1.78 MeV respectively. Above 0.9 MeV more complex levels (with two or more excited "holes") should appear.

The difference in nuclear radius between $Pb^{207}$ and $Pb^{205}$ does not cause any appreciable energy shifts of the levels, (see e.g. Bleuler *et al.* [21])). However, considerable shifts are to be expected from interactions between the three "holes" in a certain state, and also in some cases from configuration interactions. In the following paper, Pryce attempts to take into account these interactions for his calculations of the expected energy levels.

It is interesting to compare our decay scheme with his calculations, but a few words of caution might not be out of place here: 1) Only a small fraction of the levels expected from theory can be found by the methods we have used, i.e. only those levels which can be fed, directly or indirectly, by electron capture. 2) Only for a few of the levels actually found could a reasonably certain assignment be given. 3) At best, the experiments can give the spin and parity assignments, but they cannot give the actual configuration responsible for the level. However, the type of experiment used by Harvey [5]) and possibly also branching ratios for de-excitations, might indicate that a certain level is a single particle (or "hole") level.

With these reservations, the agreement between the general trend of our decay scheme and the calculations of Pryce seems to be fairly good. Thus, the assignment of $f_{5/2}$ to the ground state, the destruction of the metastability of the $i_{13/2}$ level, the absence of high spin states ($> 5/2$) at energies below 700 keV and the odd parity of all the levels below 1700 keV are some of the more striking features observed experimentally and also predicted by Pryce's calculations.

Some points of disagreement seem to be the following: 1) No $p_{1/2}$ level was found near 200 keV as expected from the calculations. However, this might not be unreasonable. Direct electron capture to such a level is expected to be negligible (spin change $= 4$), while the strongly fed levels would de-excite preferably to the ground state on account of their high spins (7/2, 9/2, 11/2). To explain why the $p_{1/2}$ state is not fed from the $p_{3/2}$ state at 282.3 keV, one has to assume that the distance between these two levels is very small (say $< 50$ keV), thus causing the $p_{3/2}$ level to de-excite predominantly to the ground state. 2) We preferred to assign spin 9/2−, rather than 7/2− to the 703.3 level (see section 3.5), though this assignment is not completely certain.

## 5. Suggestions for Future Work

The work described in this paper is by no means the last word about the level structure of $Pb^{205}$. Even with the techniques used in the present work, some points can be cleared up. Using radiogenic lead (85 % $Pb^{206}$) as the target for bombardment, a reduction of interference from $Pb^{206}$ and $Pb^{207}$ can be achieved. More multipolarity assignments for the gamma transitions could be made by using external conversion, thus making deductions about absolute conversion coefficients possible.

A valuable check for the present work would be a magnetic analysis of tritons from the (d, t) reaction of $Pb^{206}$. By this technique a much better resolution than in Harvey's experiments can be achieved, and would be very useful.

The problem of the de-excitation of the $i_{13/2}$ state might be solved by looking for isotopes having half-lives of $10^{-3}-10^{-6}$ sec (see e.g. Vegors and Axel [22])). We would expect an M2 transition of 300 keV to the 703.3 keV level if it has the spin 9/2− and an E3 transition if it has the spin 7/2−.

The present investigation has been carried out at the Nobel Institute of Physics, Stockholm, and at the Physics Dept. of the University of Uppsala.

Two of us (M. Schmorak and J. A. McDonell) would like to express our sincere thanks to Professor Manne Siegbahn, Head of the Nobel Institute, for the hospitality which has been shown to us during our stay at the Nobel Institute. M. Schmorak wishes to thank Professor Kai Siegbahn for the hospitality shown to him during his stay at the Physics Dept. of the University of Uppsala, and wants also to acknowledge the receipt of a scholarship from "Syskonen Wesséns Stiftelse" and from the Friends of the Weizmann Institute in Sweden.

The authors would like to thank Dr. H. Atterling and his cyclotron crew at the Nobel Institute for providing the necessary irradiations, Dr. K. Bleuler, Zürich, and Dr. M. H. L. Pryce, Bristol, for valuable discussions about theoretical aspects of this work, and Gerda Oelsner, B. Sc., for correcting the English.

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*Nuclear Physics* **2** (1956/57) 226—238; *North-Holland Publishing Co., Amsterdam*

# THE ENERGY LEVELS OF $Pb^{205}$

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Received 4 June 1956

**Abstract:** The energy levels of $Pb^{205}$ are calculated on the shell model, regarding $Pb^{205}$ as three holes in the double closed shell nucleus $Pb^{208}$. Using energies for single holes derived from data on $Pb^{207}$, and parameters for their pairwise interaction derived from $Pb^{206}$, the levels up to 2.6 MeV have been estimated.

## 1. Introduction

From the standpoint of the shell model, the energy levels of the isotopes of lead are particularly interesting. $Pb^{208}$ is a double closed shell, and its first excited state is correspondingly high at 2.62 MeV, since the excited states arise from the promotion of at least one nucleon to a higher shell. The spectrum of $Pb^{207}$ is very simple up to 2.34 MeV or higher, as is to be expected for a closed neutron shell with just one hole. In $Pb^{206}$ there is already more complexity, because the two neutron holes combine in many ways, but the experimentally known levels can all be classified in terms of the shell model [1]).

$Pb^{205}$, with three neutron holes is even more complex, but yet not so much so that one cannot analyze the structure of the level diagram. Going even further to the four-hole nucleus $Pb^{204}$, True [2]) has shown that calculations based on the shell model explain the main features of the decay of the isomeric 9— state. The present paper is concerned with applying to $Pb^{205}$ the ideas and methods already employed [1]) for $Pb^{206}$. The stimulus has been provided by the experimental investigation by Schmorak, Stockendal, McDonell, Bergström and Gerholm [3]) of the gamma-rays emitted by $Pb^{205}$ consequent on the electron-capture decay of $Bi^{205}$. Without detailed knowledge of the main features of the spectrum it is almost impossible to make any great progress by purely theoretical methods, because there are so many parameters which cannot be calculated accurately. But with the help of the experimental data one can be more definite.

The low-lying levels of $Pb^{205}$ arise from configurations in which three neutrons are missing from the orbitals of the top neutron shell, namely, in order of descreasing energy, $3p_{1/2}$, $2f_{5/2}$, $3p_{3/2}$, $1i_{13/2}$, $2f_{7/2}$, $1h_{9/2}$. It is probable that all the levels of $Pb^{205}$ up to 2.5 MeV are of this simple type. Above 2.5 MeV there are levels belonging to configurations in which a nucleon is excited to an orbital in the next shell. We shall call these "core-excited" levels.

## 2. The Simple Levels

The method of calculating the energy is that used in previous papers (Pryce [4]), Alburger and Pryce [1])). These will be referred to as P and AP respectively. It is assumed, as usual in the shell model, that a good approximation to a nuclear state is furnished by a single configuration. From one configuration, in general, several states of different spin $I$ can be formed. The wave function can be constructed from the wave functions of a single hole by standard methods (e.g. as given in Condon and Shortley [5])). In several of the configurations of $Pb^{205}$ some values of $I$ occur more than once, and this leads to a complication which was not present in the calculations on $Pb^{206}$. The energy of a configuration of three neutron-holes is the sum of two parts, (a) $E_0$, the sum of the energies of the three holes in the effective field of the nucleus as a whole (zero-order energy), and (b) $E_{\text{int}}$, the sum of the three pair-wise interactions of the holes (first-order interaction energy).

For the energies of the holes, moving in the nuclear well, we take the energies observed in $Pb^{207}$ from the data of Alburger and Sunyar [6]), reproduced in table I. No correction has been made for the change in nuclear radius (to the order of approximation being used the change in relative position of the orbitals is included in the interaction energy (b) with the other two missing nucleons).

TABLE I

Energy Levels of $Pb^{207}$

| | MeV |
|---|---|
| $p_{1/2}$ | 0 |
| $f_{5/2}$ | 0.569 |
| $p_{3/2}$ | 0.894 |
| $i_{13/2}$ | 1.633 |
| $f_{7/2}$ | 2.338 |
| $h_{9/2}$ | ? |

Table I differs from the corresponding table VII of AP in the identification of the 2.338 MeV level as $f_{7/2}$, instead of $h_{9/2}$.

Table II lists the more important configurations, together with their zero order energy, the spins of the levels derived from them, and the parity. Only relative energies are required, and for convenience $p^2_{1/2}\ f_{5/2}$ (the lowest configuration) has been chosen as the zero.

TABLE II

The Simple Configurations of $Pb^{205}$

| Configuration | Zero-order Energy (MeV) | Spins | Parity |
|---|---|---|---|
| $p^2_{1/2}\ f_{5/2}$ | 0 | 5/2 | − |
| $p^2_{1/2}\ p_{3/2}$ | 0.32 | 3/2 | − |
| $p_{1/2}\ f^2_{5/2}$ | 0.57 | 1/2, 3/2, 5/2, 7/2, 9/2 | − |
| $p_{1/2}\ f_{5/2}\ p_{3/2}$ | 0.89 | 1/2, $(3/2)^2$, $(5/2)^2$, $(7/2)^2$, 9/2 | − |
| $p^2_{1/2}\ i_{13/2}$ | 1.06 | 13/2 | + |
| $f^3_{5/2}$ | 1.14 | 3/2, 5/2, 9/2 | − |
| $p_{1/2}\ p^2_{3/2}$ | 1.21 | 1/2, 3/2, 5/2 | − |
| $f^2_{5/2}\ p_{3/2}$ | 1.46 | 1/2, $(3/2)^2$, $(5/2)^2$, $(7/2)^2$, 9/2, 11/2 | − |
| $p_{1/2}\ f_{5/2}\ i_{13/2}$ | 1.63 | 7/2, $(9/2)^2$, $(11/2)^2$, $(13/2)^2$, $(15/2)^2$, $(17/2)^2$, 19/2 | + |
| $f_{5/2}\ p^2_{3/2}$ | 1.78 | 1/2, 3/2, $(5/2)^2$, 7/2, 9/2 | − |
| $p^2_{1/2}\ f_{7/2}$ | 1.77 | 7/2 | − |
| $p_{1/2}\ p_{3/2}\ i_{13/2}$ | 1.95 | 9/2, $(11/2)^2$, $(13/2)^2$, $(15/2)^2$, 17/2 | + |
| $f^2_{5/2}\ i_{13/2}$ | 2.20 | 5/2, 7/2, $(9/2)^2$, $(11/2)^2$, $(13/2)^3$, $(15/2)^2$, $(17/2)^2$, (19/2), (21/2) | + |
| $p_{1/2}\ f_{5/2}\ f_{7/2}$ | 2.34 | 1/2, $(3/2)^2$, $(5/2)^2$, $(7/2)^2$, $(9/2)^2$, $(11/2)^2$, (13/2) | − |
| $p_{1/2}\ p_{3/2}\ f_{7/2}$ | 2.66 | 3/2, $(5/2)^2$, $(7/2)^2$, $(9/2)^2$, 11/2 | − |

To calculate the first-order interaction energy between the three pairs of neutron holes in a given level is straightforward when there is only one level of spin $I$ in the configuration. One just has to evaluate

$$E_{\text{int}} = \langle|V_{23}+V_{31}+V_{12}|\rangle$$

for the appropriately formed wave function $|\rangle$. This reduces to linear combinations of integrals involving two holes only [5]). Precisely these two-hole integrals occur in the calculation for $Pb^{206}$, so that the previous work can be taken over directly.

To illustrate, consider the lowest configuration, $p^2_{1/2}\ f_{5/2}$, containing only one level ($I = 5/2$). The two $p_{1/2}$ holes necessarily couple to a resultant spin of zero, because of the Pauli principle, so their mutual interaction is precisely as in the $(p_{1/2})^2 I = 0$ state of $\mathrm{Pb}^{206}$. The resultant spin when one of the $\mathrm{p}_{1/2}$ holes is coupled to the $\mathrm{f}_{5/2}$ hole can take on two values, 2 or 3. A simple calculation shows that these have weights 5/12 and 7/12 respectively. The mutual interaction of such a pair of holes is then obtained by combining the energies for the states $I = 2$, 3 of the $\mathrm{p}_{1/2}\,\mathrm{f}_{5/2}$ configuration in $\mathrm{Pb}^{206}$ with these weights. Since there are two $\mathrm{p}_{1/2}\,\mathrm{f}_{5/2}$ pairs in $\mathrm{p}_{1/2}^2\,\mathrm{f}_{5/2}$, this contribution occurs twice. I.e.

$$E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}^2\,\mathrm{f}_{5/2})_{5/2}\right] = E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}^2)_0\right] + \tfrac{5}{6}\,E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}\,\mathrm{f}_{5/2})_2\right] + \tfrac{7}{6}\,E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}\,\mathrm{f}_{5/2})_3\right].$$

The calculations simplify considerably if it can be assumed that the range of nuclear forces is short in comparison with the nuclear radius 4) (P). In analyzing the levels of $\mathrm{Pb}^{206}$ it was found that this approximation was quite adequate to correlate the data. In the present paper we shall maintain this simplifying assumption. The interaction energy of a pair of holes is then expressible in terms of one parameter for each two-hole configuration. This was denoted $-\varepsilon_s$ in P and $\varepsilon_1$, $\varepsilon_2$, . . . etc. in AP. To avoid unsightly indices, we shall here denote the interaction parameters by $a$, $b$, $c$, . . ., according to the key in table III.

TABLE III

Key to $\mathrm{Pb}^{206}$ Configuration Parameters

| $\mathrm{Pb}^{206}$ Configuration | Symbol | Value assumed (MeV) |
|---|---|---|
| $\mathrm{p}^2_{1/2}$ | $a$ | 0.35 |
| $\mathrm{p}_{1/2}\,\mathrm{f}_{5/2}$ | $b$ | 0.25 |
| $\mathrm{p}_{1/2}\,\mathrm{p}_{3/2}$ | $c$ | 0.35 |
| $\mathrm{f}^2_{5/2}$ | $d$ | 0.30 |
| $\mathrm{f}_{5/2}\,\mathrm{p}_{3/2}$ | $e$ | 0.25 |
| $\mathrm{p}_{1/2}\,\mathrm{i}_{13/2}$ | $f$ | 0.18 |
| $\mathrm{p}^2_{3/2}$ | $g$ | 0.35 |
| $\mathrm{f}_{5/2}\,\mathrm{i}_{13/2}$ | $h$ | 0.20 |
| $\mathrm{p}_{1/2}\,\mathrm{f}_{7/2}$ | $i$ | 0.25 |
| $\mathrm{p}_{3/2}\,\mathrm{i}_{13/2}$ | $j$ | 0.18 |
| $\mathrm{f}_{5/2}\mathrm{f}_{7/2}$ | $k$ | 0.30 |
| $\mathrm{p}_{3/2}\,\mathrm{f}_{7/2}$ | $l$ | 0.25 |

With this approximation (cf. P, AP) one has

$$E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}^2)_0\right] = -a,$$

$$E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}\,\mathrm{f}_{5/2})_2\right] = -\tfrac{6}{5}b,$$

$$E_{\mathrm{int}}\left[(\mathrm{p}_{1/2}\,\mathrm{f}_{5/2})_3\right] = 0,$$

whence

$$E_{\text{int}}\left[(p_{1/2}^2\, f_{5/2})_{5/2}\right] = -(a+b).$$

When a particular spin $I$ occurs twice in a given configuration, one can form two linearly independent and orthogonal wave functions $|\,1\rangle$ and $|\,2\rangle$. It is then necessary to diagonalize the matrix $E_{ij}$:

$$E_{ij} = \langle i|V_{23}+V_{31}+V_{12}|j\rangle.$$

TABLE IV

Calculated Energies

| Spin and Parity | Configuration | First-order Energy | Numerical Estimate (Zero shifted 1.00 MeV — see text) | |
|---|---|---|---|---|
| | | | Un-corrected | Corrected |
| 1/2− | $p_{1/2}\; f^2_{5/2}$ | $0.57-b-3d$ | 0.42 | 0.1 |
| | $p_{1/2}\; f_{5/2}\; p_{3/2}$ | $0.89-\frac{6}{5}b-\frac{4}{5}c$ | 1.31 | 1.4 |
| | $p_{1/2}\; p^2_{3/2}$ | $1.21-c-2g$ | 1.16 | 1.2 |
| | $f^2_{5/2}\; p_{3/2}$ | $1.46-\frac{24}{35}d-\frac{16}{35}e$ | 2.14 | 2.1 |
| | $f_{5/2}\; p^2_{3/2}$ | $1.78-\frac{3}{5}e-\frac{2}{5}g$ | 2.49 | 2.5 |
| 3/2− | $p^2_{1/2}\; p_{3/2}$ | $0.32-a-c$ | 0.62 | 0.3 |
| | $p_{1/2}\; f^2_{5/2}$ | $0.57-\frac{8}{5}b-\frac{24}{35}d$ | 0.96 | 0.7 |
| | $p_{1/2}\; f_{5/2}\; p_{3/2}$ | $\left\{\begin{array}{l}0.89-\frac{3}{5}b-\frac{2}{5}c-\frac{6}{35}e\\ \pm\frac{1}{5}\left[9b^2+4c^2+\frac{36}{49}e^2+9bc+\frac{27}{7}be+\frac{3}{7}ce\right]^{\frac{1}{2}}\end{array}\right.$ | 1.27<br>1.85 | 1.1<br>1.8 |
| | $f^3_{5/2}$ | $1.14-\frac{12}{7}d$ | 1.63 | 1.6 |
| | $p_{1/2}\; p^2_{3/2}$ | $1.21-\frac{2}{5}c-\frac{2}{5}g$ | 1.93 | 2.0 |
| | $f^2_{5/2}\; p_{3/2}$ | $\left\{\begin{array}{l}1.46-\frac{129}{70}d-\frac{23}{35}e\\ \pm\frac{1}{70}[(81d+24e)^2+1120e^2]^{\frac{1}{2}}\end{array}\right.$ | 1.29<br>2.19 | 1.4<br>2.2 |
| | $f_{5/2}\; p^2_{3/2}$ | $1.78-\frac{6}{35}e-\frac{2}{5}g$ | 2.60 | 2.6 |
| | $p^3_{3/2}$ | $2.10-2g$ | 2.40 | 2.4 |
| 5/2− | $p^2_{1/2}\; f_{5/2}$ | $0.00-a-b$ | 0.40 | 0.0 |
| | $f^3_{5/2}$ | $1.14-3\text{d}$ | 1.24 | 1.1 |
| | $p_{1/2}\; f^2_{5/2}$ | $0.57-\frac{3}{5}b-\frac{24}{35}d$ | 1.29 | 1.2 |
| | $p_{1/2}\; f_{5/2}\; p_{3/2}$ | $\left\{\begin{array}{l}0.89-\frac{3}{5}b-\frac{2}{5}c-\frac{6}{35}e\\ \pm\frac{1}{5}\left[9b^2+4c^2+\frac{36}{49}e^2+4bc-4be-\frac{8}{7}ce\right]^{\frac{1}{2}}\end{array}\right.$ | 1.32<br>1.80 | 1.3<br>1.7 |
| | $p_{1/2}\; p^2_{3/2}$ | $1.21-\frac{7}{5}c-\frac{2}{5}g$ | 1.58 | 1.5 |

| Spin and Parity | Configuration | First-order Energy | Numerical Estimate (Zero shifted 1.00 MeV — see text) | |
|---|---|---|---|---|
| | | | Un-corrected | Corrected |
| 5/2− | $f_{5/2}^2\ p_{3/2}$ | $\left\{ 1.46-\frac{17}{35}d-\frac{4}{5}e \right.$ | 1.93 | 1.9 |
| | | $\left. \pm\frac{1}{35}[49d^2+232de+304e^2]^{\frac{1}{2}} \right.$ | 2.39 | 2.3 |
| | $f_{5/2}\ p^2_{3/2}$ | $\left\{ 1.78-\frac{23}{35}e-\frac{6}{5}g \right.$ | 1.81 | 1.8 |
| | | $\pm\frac{2}{5}[(\frac{6}{7}e+2g)^2+\frac{10}{7}e^2]^{\frac{1}{2}}$ | 2.57 | 2.6 |
| 7/2− | $p_{1/2}\ f^2_{5/2}$ | $0.57-2b-\frac{2}{7}d$ | 0.98 | 0.7 |
| | $p_{1/2}\ f_{5/2}\ p_{3/2}$ | $\left\{ 0.89-\frac{3}{5}b-\frac{2}{5}c-\frac{4}{7}e \right.$ | 1.21 | 1.0 |
| | | $\pm\frac{1}{5}[9b^2+4c^2+\frac{400}{49}e^2-3bc+15be-\frac{55}{7}ce]^{\frac{1}{2}}$ | 1.71 | 1.6 |
| | $f^2_{5/2}\ p_{3/2}$ | $\left\{ 1.46-\frac{17}{35}d-\frac{4}{5}e \right.$ | 1.89 | 1.9 |
| | | $\pm\frac{1}{35}[49d^2+232de+619e^2]^{\frac{1}{2}}$ | 2.34 | 2.4 |
| | $f_{5/2}\ p^2_{3/2}$ | $1.78-\frac{46}{35}e-\frac{2}{5}g$ | 2.31 | 2.3 |
| | $p^2_{1/2}\ f_{7/2}$ | $1.77-a-i$ | 2.17 | 1.8 |
| 7/2+ | $p_{1/2}\ f_{5/2}\ i_{13/2}$ | $1.63$ | 2.63 | 2.6 |
| 9/2− | $p_{1/2}\ f_{5/2}\ p_{3/2}$ | $0.89-\frac{4}{5}c-\frac{8}{7}e$ | 1.33 | 1.2 |
| | $p_{1/2}\ f^2_{5/2}$ | $0.57-\frac{1}{5}b-\frac{2}{7}d$ | 1.43 | 1.4 |
| | $f^3_{5/2}$ | $1.14-\frac{39}{35}d$ | 1.81 | 1.8 |
| | $f^2_{5/2}\ p_{3/2}$ | $1.46-\frac{2}{7}d-\frac{32}{35}e$ | 2.15 | 2.1 |
| | $f_{5/2}\ p^2_{3/2}$ | $1.78-\frac{11}{7}e-\frac{2}{5}g$ | 2.25 | 2.3 |
| 9/2+ | $p_{1/2}\ f_{5/2}\ i_{13/2}$ | $\left\{ 1.63-\frac{3}{5}b-\frac{7}{15}f-\frac{21}{143}h \right.$ | 2.18 | 2.1 |
| | | $\pm[(\frac{3}{10}b+\frac{13}{30}f+\frac{21}{143}h)^2+\frac{3}{100}(3b+f)^2]^{\frac{1}{2}}$ | 2.56 | 2.5 |
| 11/2− | $f^2_{5/2}\ p_{3/2}$ | $1.46-\frac{2}{7}d-\frac{13}{7}e$ | 1.91 | 1.8 |
| 11/2+ | $p_{1/2}\ f_{5/2}\ i_{13/2}$ | $\left\{ 1.63-\frac{3}{5}b-\frac{7}{15}f-\frac{21}{143}h \right.$ | 2.14 | 2.1 |
| | | $\pm[(\frac{1}{15}b+\frac{19}{45}f+\frac{21}{143}h)^2+\frac{16}{405}(3b+f)^2]^{\frac{1}{2}}$ | 2.60 | 2.6 |
| 13/2+ | $p^2_{1/2}\ i_{13/2}$ | $1.06-a-f$ | 1.53 | 1.1 |
| | $p_{1/2}\ f_{5/2}\ i_{13/2}$ | $\left\{ 1.63-\frac{3}{5}b-\frac{7}{15}f-\frac{336}{1105}h \right.$ | 2.16 | 2.1 |
| | | $\pm[(\frac{9}{70}b-\frac{89}{210}f+\frac{336}{1105}h)^2+\frac{187}{4900}(3b+f)^2]^{\frac{1}{2}}$ | 2.52 | 2.5 |
| | $f^2_{5/2}\ i_{13/2}$ | $2.20-3d-h$ | 2.10 | 2.1 |
| 15/2+ | $p_{1/2}\ f_{5/2}\ i_{13/2}$ | $\left\{ 1.63-\frac{3}{5}b-\frac{7}{15}f-\frac{336}{1105}h \right.$ | 2.16 | 2.1 |
| | | $\pm[(-\frac{3}{10}b+\frac{13}{30}f+\frac{336}{1105}h)^2+\frac{3}{100}(3b+f)^2]^{\frac{1}{2}}$ | 2.51 | 2.5 |

These matrix elements are similarly combinations of interaction energies occurring in $\mathrm{Pb}^{206}$, and the evaluation of the energy is again straightforward, though tedious.

The result of these calculations is given in table IV, column 3. Column 4 lists the numerical values calculated using the parameters in column 3 of table III. The zero has been shifted downwards 1.00 MeV as explained below.

## 3. Configuration Interaction

The starting point of the calculations has been the assumption that a shell-model configuration is a good approximation to the nuclear state. When states of the same spin and parity are close together in energy this will no longer be true, and there will be mixing of configurations. It would no doubt be possible to calculate the effect of this configuration interaction on all the levels by second-order perturbation methods, but the labour would be quite considerable, and the basic accuracy of the method is probably not high enough to justify it. Instead, a rough estimate of the energy shift has been made for a few of the more important levels, and the rest have been corrected by guess-work and intuition. The energies calculated assuming plausible values of the parameters, and corrected in this way for configuration interaction, are listed in table IV, column 5.

## 4. Numerical Values

The parameters have been estimated from the energy levels of $\mathrm{Pb}^{206}$, combined with a few rough calculations using assumed radial wave-functions, to estimate the relative values of the mutual attraction of two p-holes, or two f-holes, or of a p-hole and an f-hole, etc. They are listed in the third column of table III.

The energy shift due to configuration interaction is by no means negligible in some levels. It is particularly important in those levels where two holes in identical orbitals combine their angular momenta to zero resultant (e.g. $(\mathrm{p}^2_{1/2}\ \mathrm{f}_{5/2})_{5/2}$, $(\mathrm{f}^3_{5/2})_{5/2}$, $(\mathrm{p}^2_{1/2}\ \mathrm{p}_{3/2})_{3/2}$ etc.). In these levels the diagonal matrix element of the mutual attraction between holes is large; and the off-diagonal elements between similar states are correspondingly large. For instance it can be estimated that the matrix element of $V_{\mathrm{int}}$ linking the ground state $(\mathrm{p}^2_{1/2}\ \mathrm{f}_{5/2})_{5/2}$ with $(\mathrm{f}_{5/2})^3{}_{5/2}$ is $b\sqrt{2}$ (in the short range approximation).

According to second order perturbation theory these two states push each other apart by an amount $\approx 2b^2/\Delta E$ ($\Delta E$ being their energy difference). This comes to about 0.15 MeV. Similarly the perturbation of $(p_{1/2}^2\ f_{5/2})_{5/2}$ by $(p_{3/2}^2\ f_{5/2})_{5/2}$ comes to about 0.15 MeV. (The matrix element is $c\sqrt{2}$). Considering all of the levels which perturb the ground state, we estimate that altogether it is pushed down 0.40 MeV by configuration interaction. All the states of the same type $p_{1/2}^2 j$ are similarly lowered by about the same amount.

In column 3 of table IV, the relative energy of the ground state, uncorrected for configuration interaction, is listed as $0.00-a-b$, which gives $-0.60$ MeV on substituting numerical values. Adding the further correction $-0.40$ MeV for configuration interaction yields the relative energy $-1.00$ MeV. Since it is convenient in the end to list energies relative to the ground state, all energies in columns 4 and 5 have been raised by 1.00 MeV to bring the corrected ground state to 0.00 MeV.

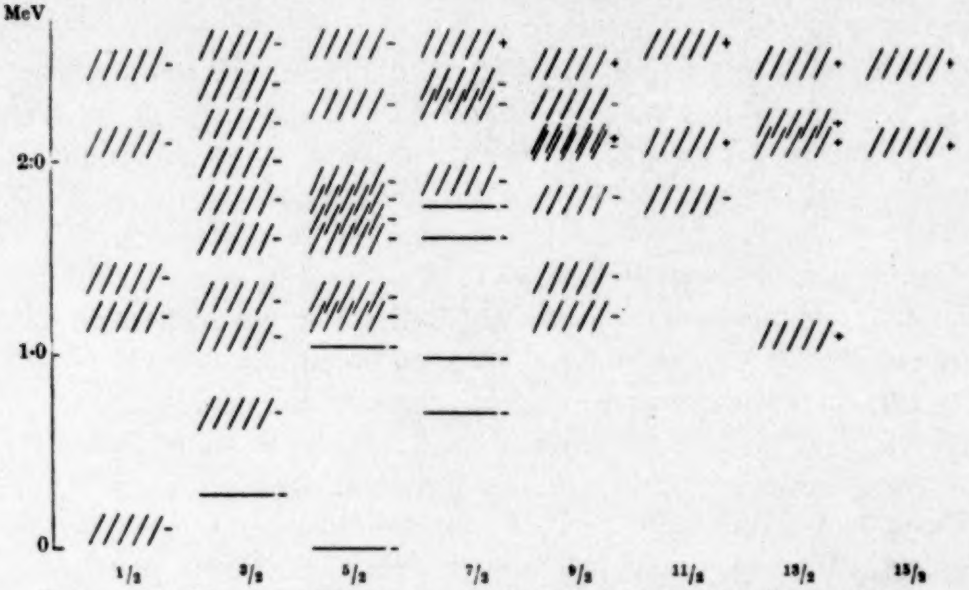
The numerical values of the interaction parameters in table III are somewhat smaller than the value in table IX of AP. The reason is that in AP inadequate correction was made for configuration interaction, particularly on the $(p_{1/2}\ f_{5/2})_2$ level, from which the value of $b$ was deduced. The present values, however, are still only very rough, and could probably be somewhat improved by correlating the experimental data on $Pb^{205}$ and $Pb^{206}$ with more complete calculations. On the other hand the approximations have probably already been pushed further than they are valid, and it could very well be that further numerical refinements would only be misleading.

## 5. Core-Excited Levels

The lowest core-excited level of $Pb^{205}$ can plausibly be assumed to be similar in structure to the (3—) first excited level of $Pb^{208}$, which is most probably a state in which a proton is excited to the next shell (to $h_{9/2}$ or $f_{7/2}$, leaving a hole in $s_{1/2}$ or $d_{3/2}$). In $Pb^{205}$ this proton configuration combines with the $p_{1/2}^2\ f_{5/2}$ (neutron hole) configuration. There are very many resultant levels, all of even parity, with spins ranging from 1/2 to high values. To calculate their energy would be very difficult at this stage. One may estimate, however, that it is not very far (say 0.4 MeV) from 2.6 MeV, the energy of the 3— level in $Pb^{208}$.

## 6. Structure of the Level Diagram

The general structure of the predicted energy diagram of Pb$^{205}$ emerges clearly from these calculations. It is shown diagramatically in fig. 1. The energy levels are there separated according to their spin. The parity of the level is indicated by the appropriate symbol alongside. Those for which a reasonable identification can be made with experimentally established levels are indicated by horizontal lines, while the others, to give some indication of the uncertainty of the calculated energy, are shown by hatching. It should be emphasized, however, that the variation shown is less than the real uncertainty, in order not to confuse the figure. The uncertainty may well amount to ±0.2 MeV, particularly in those levels where two nucleons are paired off to zero resultant, with consequent high mutual interaction (e.g. $(p_{1/2}\ f^2_{5/2})_{1/2}$).



The predicted levels of Pb$^{205}$ up to 2.6 MeV (core-excited levels not shown).

Although the experimental data slightly favour 9/2 — rather than 7/2 — for the 703.3 keV level, the calculations definitely indicate that 7/2 — lies below 9/2 —. Fig. 1 therefore shows this level as 7/2 —.

Core excited levels have not been indicated.

## 7. The Ground State of Pb$^{205}$

From the calculations alone it would be difficult to be sure whether the spin of the ground state is 5/2— or 1/2— (i.e. from the configurations $p^2_{1/2}\ f_{5/2}$ or $p_{1/2}\ f^2_{5/2}$), although the former appears the more probable. The experimental data are easier to explain if the ground state is 5/2—. In the first place, $Pb^{205}$ is unstable, by an estimated 370±100 keV (Shaw and Prescot [7]), against electron capture to $Tl^{205}$, but its radioactivity has not so far been observed†. This could be explained if the transition were strongly forbidden, and so long-lived. Since the spin of $Tl^{205}$ is 1/2, this indicates $I \geqq 5/2$ for the ground state of $Pb^{205}$. Again, the strong (probably M1) gamma-ray of 1766 keV emitted by $Pb^{205}$ following electron capture by $Bi^{205}$ seems to be best explained by a transition from a 7/2 level to the ground state (see below), and this also implies $I \geqq 5/2$. The absence of appreciable direct capture into the ground state in the decay of $Bi^{205}$ (9/2—) indicates $I \leqq 5/2$. Taking both these conditions together leads to the conclusion that the spin is indeed 5/2.

## 8. The Excitation of $Pb^{205}$ in the Decay of $Bi^{205}$

As the major experimental information concerning the energy levels of $Pb^{205}$ comes from the gamma rays emitted following the decay of $Bi^{205}$, it is relevant to consider which levels of $Pb^{205}$ are directly excited.

As a first step we must consider the ground state of $Bi^{205}$. Presumably this is 9/2— ($h_{9/2}$ proton, with four neutron holes in $p_{1/2}^2\, f_{5/2}^2$). The normal selection rules for electron capture favour transitions in which $\Delta I = 0$ or $\pm 1$, unless there are special factors involving nuclear matrix elements. In the present case, one proton must change into a neutron. Capture into the ground state of $Pb^{205}$ is possible by the conversion of the $h_{9/2}$ proton into an $f_{5/2}$ neutron, but this implies $\Delta I = 2$ (no change of parity) and is thus a second-forbidden transition. Capture into states with $I = 7/2$, 9/2 or 11/2 satisfies the $\Delta I$ condition, but involves considerable internal rearrangement, and will therefore be slow compared with a straightforward allowed transition. Conversion of a proton from an inner shell into a $p_{1/2}$ or $f_{5/2}$ neutron can lead to a core-excited state, if enough energy is available. For instance a $d_{3/2}$ proton capturing an electron to become an $f_{5/2}$ neutron will leave the $Pb^{205}$ in one of the core-excited states postulated around 2.6 MeV.

Taking these three possibilities in turn, we can make the following predictions:

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† *Note added in proof:* Huizenga and Wing [9]) report decay of $Pb^{205}$ by L-capture with half-life $5 \times 10^7$ years.

(i) Capture into the ground state is doubly forbidden, with partial log *ft*-value $> 8$ (based on a disintegration energy of 2.6 MeV).

(ii) Capture is to be expected into all states of $I = 7/2$, $9/2$, $11/2$ below 2.6 MeV, but with log *ft* values considerably in excess of 5. It may be asserted that since the ground state of $\mathrm{Bi}^{205}$ must contain a considerable (say 6 %) admixture of the neutron-hole configuration $(\mathrm{p}_{1/2}^2\ \mathrm{f}_{7/2}^2)_0$, coupled with the $\mathrm{h}_{9/2}$ proton state, capture can lead fairly easily to the $(\mathrm{p}_{1/2}^2\ \mathrm{f}_{7/2})_{7/2}$ state of $\mathrm{Pb}^{205}$, since this involves the transformation of the $\mathrm{h}_{9/2}$ proton from this admixture into an $\mathrm{f}_{7/2}$ neutron — a transition in which the angular momentum of a single nucleon changes by one unit. Although the transition $\mathrm{h}_{9/2} \rightarrow \mathrm{f}_{7/2}$ is *l*-forbidden, it is plausible that this competes favourably with all others, for they involve more complicated admixtures of configurations. We may thus expect the $\mathrm{p}_{1/2}^2\ \mathrm{f}_{7/2}$ level to be strongly populated in the decay of $\mathrm{Bi}^{205}$. Since this can be estimated rather precisely to lie near 1.8 MeV (the interaction energies are nearly identical in $\mathrm{p}_{1/2}^2\ \mathrm{f}_{7/2}$ and $\mathrm{p}_{1/2}^2\ \mathrm{f}_{5/2}$, so that the energy difference is close to that between $\mathrm{f}_{7/2}$ and $\mathrm{f}_{5/2}$ in $\mathrm{Pb}^{207}$, namely 1.77 MeV), it is tempting to identify the strong 1766.3 keV gamma as a transition between these levels.

(iii) Conversion of an internal $\mathrm{d}_{3/2}$ proton of $\mathrm{Bi}^{205}$ to a $\mathrm{f}_{5/2}$ neutron of $\mathrm{Pb}^{205}$ corresponds to a first forbidden transition of the type which competes strongly with allowed transitions in heavy elements ($\Delta I = 1$, yes) and can lead to core excited levels of spin $7/2$, $9/2$, $11/2$, if any of these lie below 2.6 MeV (the transition energy). Branching ratios will depend strongly on the available energy. The 2.6 MeV gamma ray may correspond to such a transition. It would then be an E1 radiation, from $7/2+$ to $5/2-$. An analogous situation is encountered in the decay of $\mathrm{Bi}^{206}$, which goes predominantly to two levels of $\mathrm{Pb}^{206}$ which are estimated to have a strong core-excitation character (AP), in preference to other lower levels of spin allowed by the selection rules.

## 9. Possible Isomerism

The isotopes 197, 199, 201, 202, 203, 204, 206, 207 of lead all have isomeric levels, and it is interesting to consider whether this is the case also in $\mathrm{Pb}^{205}$. The only level which could be isomeric is $(\mathrm{p}_{1/2}^2\ \mathrm{i}_{13/2})_{13/2}$. If we define an isomeric transition by the condition

If 13/2+ is isomeric, it should emit E3 radiation of about 400 keV to the 7/2− level at 703.3 keV. This implies a life-time around a millisecond [8]). The absence of such radiation in the gamma spectrum of $Bi^{205}$ is not a conclusive argument against the isomeric nature of the 13/2+ level, for its excitation in the decay of $Bi^{205}$ might be very weak and escape detection.

## 10. Conclusions

It will be seen from fig. 1 that the simple theory based on the shell model makes fairly definite predictions about the position of the levels of spins from 1/2 to 15/2. Above 1 MeV, for 3/2, 5/2 and 7/2, the level density is high, about five levels per MeV. Since a typical matrix element for configuration interaction is 0.1 MeV, considerable mixing and shifting of these levels is to be expected, but not yet to the extent that their qualitative distribution is changed. Where two levels are calculated to come so close together that no unique configuration can be assigned to either separately, this has been indicated in table IV by bracketing.

If one continued to apply this theoretical approach to lighter isotopes of lead, one would soon reach a stage where, for all but the very lowest levels, the level spacing would be smaller than the matrix elements of configuration interaction. Second-order perturbation methods would then become inadequate, and one might anticipate that strong collective effects would set in. In the present calculations we have completely ignored collective effects. This is justified by the detailed calculations of True [2]), who showed that they are small even in $Pb^{204}$.

In spite of the fact that $Pb^{204}$ has four holes, as against three in $Pb^{205}$, the level diagram predicted by True for $Pb^{204}$ is not noticeably more complex than the level scheme predicted here for $Pb^{205}$. This illustrates a general distinction between the spectra of even and odd mass nuclei. In even nuclei, the 0+ ground state is strongly depressed by pairing effects, so that the complicated features of the spectrum start at a higher excitation relative to ground, and the structure of the lower levels appears more simple.

Table IV shows clearly the strong effect of pairing two nucleons to a zero resultant. Such levels are distinctly lower in energy than the others from the same configuration. One example is $(p_{1/2}\, f^2_{5/2})_{5/2}$, which is 0.6 MeV below the next lowest level of the

$\Delta I \geqq 3$, reference to fig. 1 shows that the 13/2 level is isomeric provided it lies below the lowest 9/2 level. Estimated energies are 13/2+ : 1.1 MeV and 9/2− : 1.2 MeV. Within the theoretical error of $\pm 0.2$ MeV it is thus impossible to be certain.

configuration. Another, less obvious, example is the 3/2 level at 1.3 MeV from $f^2_{5/2}\ p_{3/2}$, which is 0.6 MeV lower than the other levels. There are two 3/2 levels in this configuration, neither of which correspond exactly to pairing off $f^2_{5/2}$ to zero resultant. A study of the detailed calculations shows, however, that the weights of $(f^2_{5/2})_0$, $(f^2_{5/2})_2$ and $(f^2_{5/2})_4$ in this level are respectively 98 %, 2 % and 0, so that it is predominantly of the paired-off type. This observation enables one to estimate sufficiently accurately the energy of other similar levels, such as $(f^2_{5/2}\ i_{13/2})_{13/2}$, without diagonalizing the full interaction matrix.

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# MAGNETIC PROPERTIES OF $F^{19}$ IN THE 197 keV NUCLEAR LEVEL

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Received 6 July 1956

**Abstract:** Gamma rays are observed from the de-excitation of the second excited state of $F^{19}$ produced by inelastic proton scattering. It is found that anisotropy of this radiation is quenched by a uniform magnetic field applied normal to the proton beam, and a measurement of this effect yields the following value for the nuclear spectroscopic splitting factor: $g_I = 1.8 \pm 0.4$. The degree of anisotropy cannot be influenced by a field applied parallel to the proton beam in the case of solid NaF and $MnF_2$ targets, but with a gas target of $CF_4$ such a field restores the undisturbed distribution. This effect is analysed with the aid of energy loss data and implies as an order of magnitude a hyperfine structure constant $a = 0.2 \times 10^{-18}$ erg for the free $F^{19}$ in its ground $P_{3/2}$ state.

## 1. Introduction

It was first pointed by Goertzel [1]) that a nuclear level of finite lifetime can be influenced by magnetic fields due to extranuclear electrons in such a way that the natural anisotropy of nuclear radiations is reduced. This effect should be important with inverse nuclear lifetimes as large as the hyperfine structure frequency splitting, or less. More recently it has been shown possible in a number of ways to influence nuclear radiations with artificially produced magnetic fields and electric field gradients, and thus measure the appropriate nuclear moments. Experimental results have been summarised by Steffen [2]), particularly for disturbed gamma-gamma correlations.

The experiments described here depend on the same principle. The 197 keV level of $F^{19}$, of spin $I = \frac{5}{2}$, is excited by inelastic proton scattering [3]) at a resonance energy of 873 keV, and the angular distribution of the subsequent 197 keV radiation observed with respect to the direction of incoming protons. This level has a mean lifetime of [4]) $(1.24 \pm 0.07) \times 10^{-7}$ sec, sufficiently long to make it possible to influence the distribution with fields of order 1000 Oersted. In the measurements described in Section 3 a magnetic field was applied normal to the proton beam to study quenching of the initial anisotropy and hence deduce a value for the nuclear spectroscopic splitting factor $g_I$. The experiments of Sections 4 and 5 depend on the ability of a strong field to decouple magnetic interactions between the nucleus and electrons, a process similar to the Back-Goudsmit effect of optical spectroscopy. The degree of decoupling governs the resulting anisotropy in a calculable way and, provided one can estimate all other influences present, yields a value for the atomic hyperfine structure constant $a$.

The effects taking place in these processes can be made intelligible with the aid of the vector coupling model for angular momenta, as has been emphasised by Steffen [2]). In the cases of magnetic coupling, one associates the classical Larmor frequency of precession of an angular momentum vector, with the difference between two adjacent quantum energies, and this is why the nuclear $g_I$, rather than magnetic moment (proportional to $g_I I$), enters the calculations. The model also helps to explain qualitatively the connection between spatial nuclear orientation and anisotropy of nuclear radiations: in particular a static isotropic H.F.S. interaction cannot completely destroy a given nuclear spin polarization, and this leads one to expect a "hard core" of anisotropy which cannot disappear. This effect was first described by Alder [5]) and the theory has been extended by Abragam and Pound [6]) to include other interactions, particularly those expected in liquids. The basic theory applies to the present experiments, though these deal ostensibly with triple correlations. The necessary proof is sketched in Section 6, together with details of formulas used in Sections 3 to 5.

## 2. Experimental Methods

For detection of the soft 197 keV radiation a photomultiplier, mounted on a turntable, was used with a 1 inch diameter × $\frac{1}{8}$ inch NaI(Tl) crystal, and the spectrum was recorded on a kicksorter. The yield was monitored with another photomultiplier and larger crystal, set to detect 6 to 7 MeV gamma-rays from the competing reaction $F^{19}(p, \alpha\gamma)O^{16}$.

Fig. I shows in elevation schematic arrangements for observation using solid targets with magnetic field (a) normal and (b) parallel to the proton beam. The monitor counter is not shown. The magnet was supported by a separate frame and was of conventional construction, except that provision was made for removing the poles so that positions (a) and (b) may be interchanged easily. The poles were shaped as shown for maximum field intensity. In (a) the proton beam was led through a shield of steel tube to a thin target support of beryllium copper separated from the magnet by a short length of quartz tubing, and situated at the geometrical centre of the magnetic field. In position (b) the shield tube conveniently formed a core for one magnet pole. Here is was essential that no normal component of field be present, so the target was set as shown,

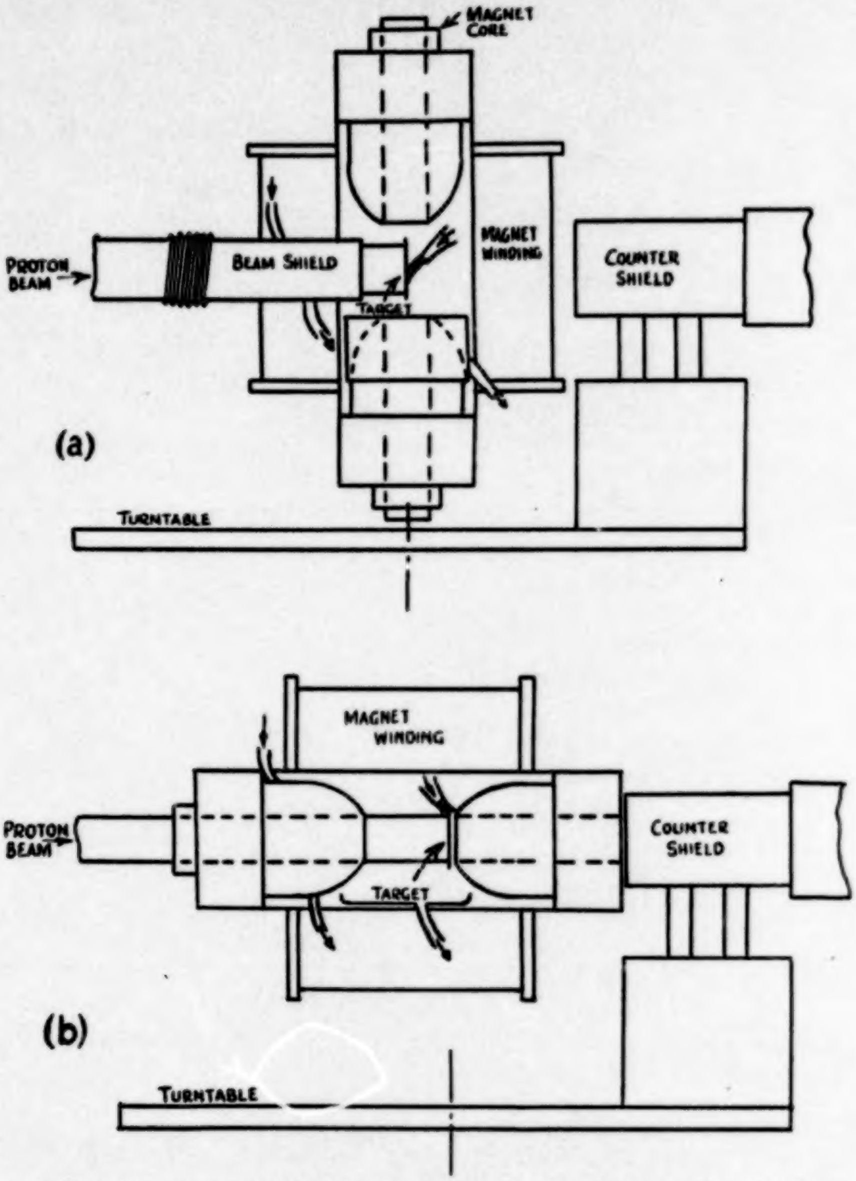
MAGNET CORE
PROTON BEAM
BEAM SHIELD
MAGNET WINDING
COUNTER SHIELD
TARGET
(a)
TURNTABLE
MAGNET WINDING
PROTON BEAM
COUNTER SHIELD
TARGET
(b)
TURNTABLE

Fig. 1. Schematic arrangement for observation with magnetic field (a) normal, (b) parallel to the proton beam. Directions of cooling water flow are shown by solid arrows.

close and parallel to one pole. In each arrangement the magnet gap was 1 inch. For experiments with gas targets a somewhat more elaborate arrangement was necessary; the target assembly will be described elsewhere. Due to the relatively large target volume (about 1 cm$^3$), it was necessary to have a more uniform magnetic field, and here the magnet poles were unshaped, of 3 inch diameter and with 1½ inch gap.

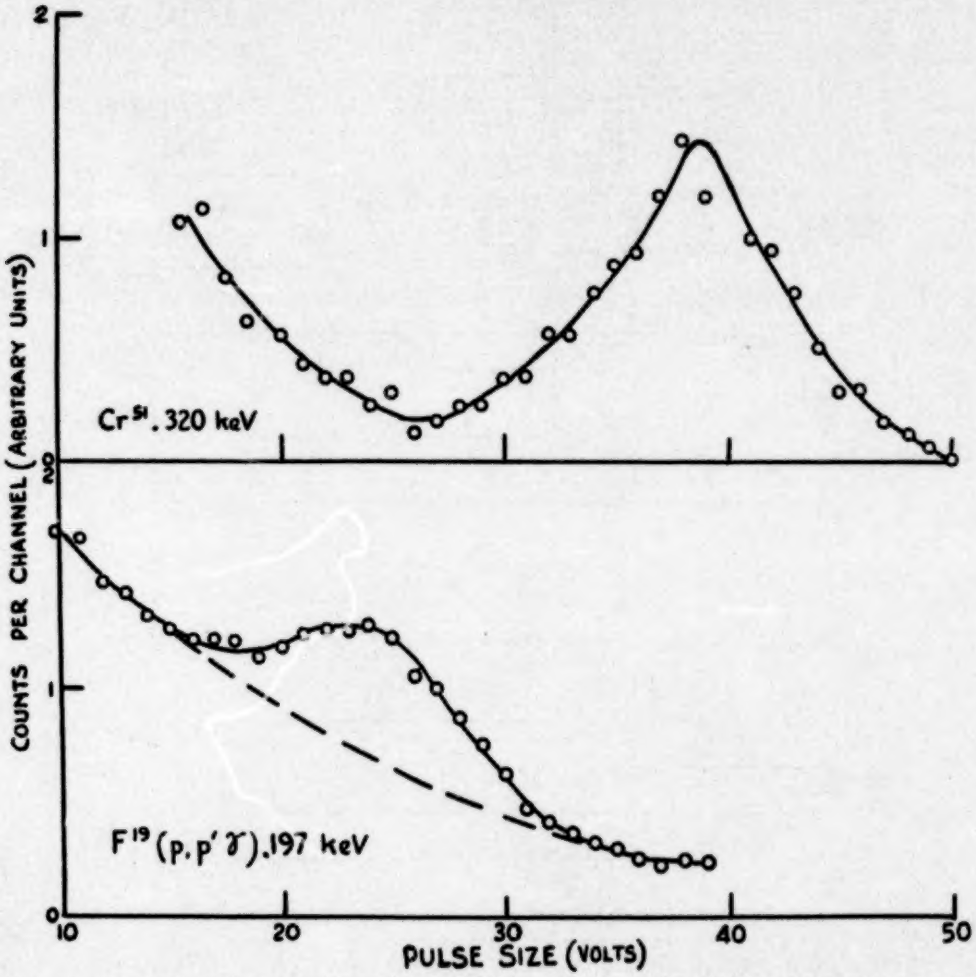

Fig. 2. Typical pulse spectrum from $F^{19}$ (p, p′ $\gamma$) and calibration spectrum of $Cr^{51}$ gamma-rays.

Steel and mumetal shields were provided for the photomultipliers in the different arrangements and effects of magnetic influences on pulse spectra carefully excluded. The angles at which detection was possible in position (b) were restricted by this requirement, as will be discussed in Section 4. The magnetic field was measured with a search coil and ballistic galvanometer previously calibrated using a standard mutual inductance. The counter was calibrated with $Cr^{51}$ and $Cs^{137}$ gamma-rays of energies 320 and 660 keV respectively. The main difficulty in performing these experiments lay in observing soft radiation in the presence of intense scattered radiation from 6 to 7 MeV gamma-rays, due to large masses of steel in the magnet poles, etc. Typical pulse spectra from $F^{19}(p, p'\gamma)$ and $Cr^{51}$ are shown in fig. 2. In each case the photo-peak is shown. The amount of background in the lower curve was judged by eye. In all the experiments described below, intensities were estimated by measuring areas under the curves, as shown in fig. 2.

## 3. Nuclear Moment Measurements †

Protons of 873 keV were allowed to strike a 30 keV polycrystalline target prepared by evaporation of NaF. This substance forms ionic crystals which should be suitable for observations on the undisturbed nuclear moment. Christy [7]) has shown from the known spin parameters that the undisturbed radiation must be distributed as

$$W(\theta) = 1+A\cos^2\theta,$$

where $\theta$ is the laboratory angle of observation and $0.23 \leqq A \leqq 0.75$. Fig. 3(a) shows the observed angular distribution to have $A = 0.42\pm0.02$. With magnetic field in position (a) and directed so as to produce spin precession in a sense opposite to that of the angle $\theta$, values were observed of $W(0°)$ and $W(100°)$. The results are shown in fig. 4, together with the theoretical form expected for the value

$$g_I = 1.1\pm0.2$$

for the nuclear spectroscopic splitting factor as usually defined (cf. for example Ramsey [8])). We have modified the value quoted in our previous communication in accordance with the most recent lifetime measurement [4]).

It must be noted that should the coefficient of anisotropy $A$ measured be less than the "true" one, the value obtained for $g_I$ must be too small by a factor which depends on the error of the apparent value of $A$. This will be discussed in Section 7 in connection with the results of an experiment similar to ours by Lehman *et al.* [9]), where a different value of $A$ seems to be implied.

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† A summary of the experiments in Sections 3 and 4 has already been described in ref. [6a]).

## 4. Hyperfine Structure in Solid Targets

In order to analyse the data of fig. 4, it was desirable to know what extranuclear interactions were present in the solid target.

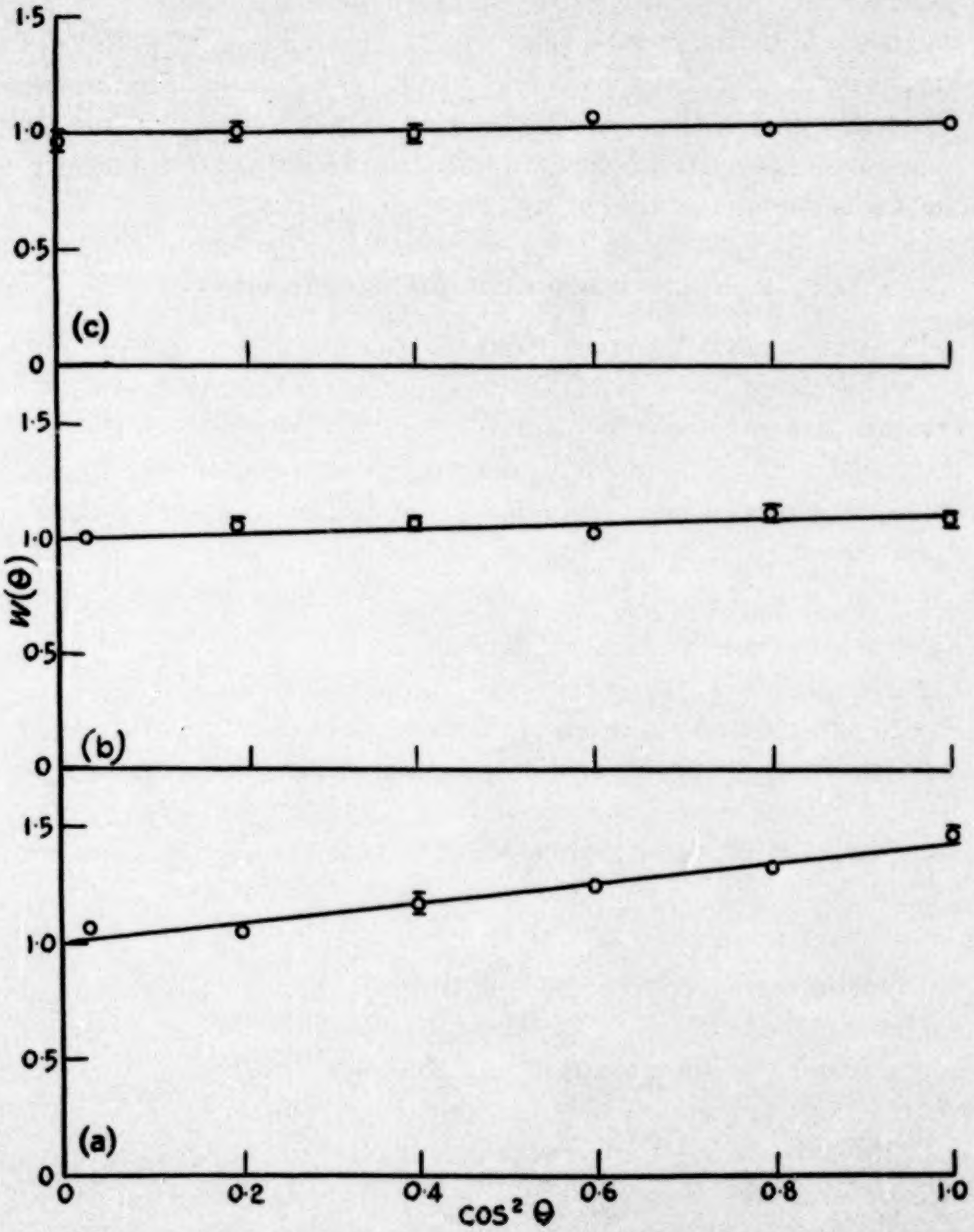

Fig. 3. Angular distributions $W(\theta)$ of 197 keV gamma-rays from (a) solid NaF, (b) solid $MnF_2$, (c) $CF_4$ gas targets. Probable errors are shown where significant.

Following the theoretical discussion of Abragam and Pound [6]), the following interactions must be considered possible:

(i) static isotropic H.F.S.;

(ii) isotropic H.F.S. with paramagnetic relaxation;
(iii) static anisotropic H.F.S.;
(iv) quadrupole interactions.

Goertzel [1]) first showed that one can test for the existence of type (i) by producing the Back-Goudsmit effect, i.e. by decoupling nuclear and electron spins with an applied field along a preferred axis. Such a field could also, under certain conditions, decouple completely with type (ii), partially with (iii) but not at all with (iv) [6]).

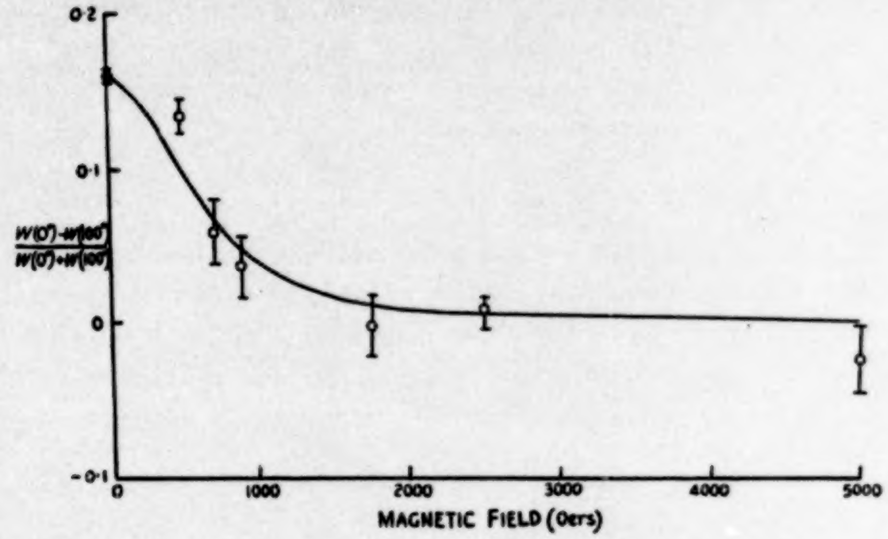

Fig. 4. Anisotropy of gamma-rays from solid NaF target as a function of magnetic field strength normal to the proton beam (position (a) of fig. 1).

A decoupling experiment was attempted using position (b) of fig. 1. Because the magnet was found to influence the operation of the counter when the latter was brought too close, this experiment was done at a fixed position $\theta = 100°$. The fractional change of counting rate at this angle can be shown to be approximately minus a quarter of that in $A$. The following result was obtained:

NaF: Decoupling field 7000 Oe. Change $(-1 \pm 3)\%$.

A change of $A$ to the maximum possible (0.75) would have required a change of $-8\ \%$ in counting rate.

A target composed of the paramagnetic salt $MnF_2$, for which (fig. 3b) the anisotropy was $A = 0.10 \pm 0.02$, gave the following result:

$MnF_2$: Decoupling field 7000 Oe. Change $(1 \pm 2)\%$.

Had this measurement yielded a restoration to the value $A = 0.42$

obtained with NaF, it would have provided circumstantial evidence that the latter represented the "true" anisotropy. Conversely, the fact that the reduced anisotropy could not be changed implies that interactions of types (ii) to (iv) must be considered, and they cannot be ruled out in the case of the NaF target. It would not be surprising to find quadrupole interactions even in an ionic crystal, as the recoiling nucleus would move far from its original site. Furthermore, near the surface of a thin target one might expect to find strong electric field gradients. †

## 5. Hyperfine Structure in the Free $F^{19}$ Atom

A study was made of the anisotropy of radiation emitted from a target of $CF_4$ gas at a pressure of 2.5 cm Hg. The angular distribution, shown in figure 3c, is virtually isotropic, with $A = 0.04 \pm 0.02$. Such a reduction of anisotropy cannot be due to H.F.S. with a single electron, as the "hard core" attenuation coefficient $G_2$ in the expansion

$$W(\theta) = 1+G_2 B_2 P_2 (\cos\theta)$$

is respectively 0.83 and 0.40 for electron spin $\frac{1}{2}$ and $\frac{3}{2}$. However, we would expect that with each nucleus, recoiling in the gas target with a mean kinetic energy of 70 keV, some hundreds of electrons would be successively captured and lost by collisions with gas molecules. As a result, the anisotropy would be completely destroyed in the absence of an applied field.

When a decoupling field was applied, it was found that the 197 keV radiation became anisotropic. The results are plotted in fig. 5. These readings were obtained with the detector set at 90° to the beam with the magnet on and off, and the quantity

$$-\frac{A}{3} = \frac{W(90^\circ)_{\mathrm{on}} - W(90^\circ)_{\mathrm{off}}}{W(90^\circ)_{\mathrm{on}}}$$

obtained, thence $G_2 B_2 = 2A/(3+A)$ plotted.

Fig. 5 cannot be interpreted without knowing how many electrons were captured and lost per nucleus. This was obtained by first estimating the stopping power in the gas, then calculating the velocity of the particle as a function of time and finally recalculating the partial cross section for charge exchange processes. We have used experimental data of Weyl[11]) for $Ne^+$ ions in $N_2$, extrapolated to lower velocities in accordance with the theory of Fermi and Teller [12]), disregarding variations in the exact charge of the recoiling nucleus (cf. ref. [13]). We have included the effect of nuclear scattering [14]) which did not contribute to Weyl's results, and used experimental data [15]) on charge exchange by Ne atoms and ions in Ne. For recoil velocities below $10^7$ cm/sec it was assumed [16]) that charge exchange occurs primarily between $F^{19}$ atoms and negative ions, of ionization energy 4 eV. After taking into account the finite nuclear lifetime, the results in table 1 were obtained:

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† In the case of $MnF_2$ interactions of type (ii) to (iv) could occur even in perfect crystals. In particular (ii) would be favoured as the paramagnetic relaxation time is very short [10]). Types (iii) and (iv) would also both be expected if the $F^{19}$ nucleus occupied a lattice site, because of the tetragonal symmetry. Elaboration of these points is not relevant to the present discussion.

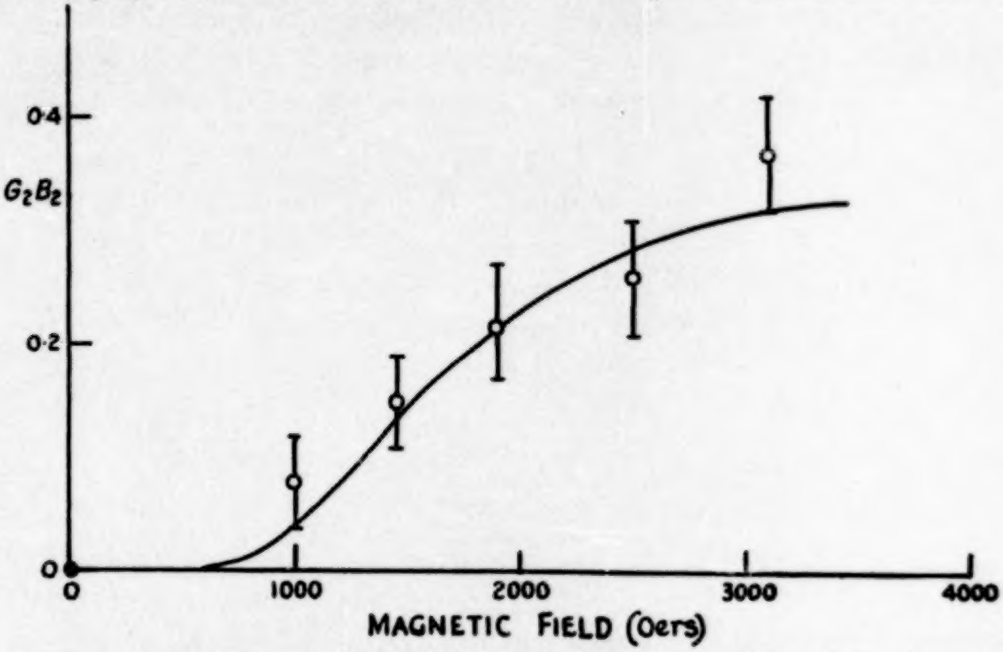

Fig. 5. Coefficient $G_2B_2$ of $P_2$ (cos $\theta$) in the expansion of $W(\theta)$ as a function of field strength parallel to the proton beam (position (b) of fig. 1), with a $CF_4$ gas target.

TABLE 1

Results of Calculations on Electron Exchange

| Recoil Velocity of Nucleus (cm/sec) | Time since Formation (sec) | No. ($n$) of Electrons Exchanged since Formation | Mean Time Spent as a $F^{19}$ Atom (sec) |
|---|---|---|---|
| $0.9 \times 10^8$ | 0 | 0 | $0.6 \times 10^{-11}$ |
| $0.1 \times 10^8$ | $0.3 \times 10^{-7}$ | 200 | $5.0 \times 10^{-11}$ |
| $0.03 \times 10^8$ | $\gg 10^{-7}$ | 250 | $17 \times 10^{-11}$ |

These values seem to be consistent with range-energy data of Evans *et al.* [17]), though we cannot regard the result as better than an order of magnitude estimate.

For an electron effectively to influence the angular distribution, its classical Larmor frequency of precession must be comparable to the inverse of the figure quoted in the last column of table 1. These two quentities are in fact equal for $F^{19}$ at $2\times10^{10}$ sec$^{-1}$ with velocity $0.1\times10^{8}$ cm/sec and magnetic field 1850 Oe, so some of the 250 electrons quoted in table 1 are ineffective. Choosing $n = 150$ we find for the H.F.S. constant

$$a = 0.22\times10^{-18} \text{ erg},$$

the curve for which is plotted in figure 5. An almost identical curve would be reproduced for $n = 250$ with $a = 0.17\times10^{-18}$ erg, so the result is relatively insensitive to changes of $n$. If anything, the estimate of $n$ is somewhat too small, as other interactions such as optical excitation, could increase it.

## 6. Theory

### (a) GENERAL CASE OF THE DISTURBED (p, p′ γ) CORRELATION

The basic theory of disturbed gamma-gamma correlations is well-known [1, 5, 6]). In the treatment of particles, one merely replaces the irreducible tensor operators $D^{L}_{M\pm1}$ defined, for example, by Goertzel [1]), by the spherical harmonics

$$D^{L}_{M0} = Y^{M}_{L}(\Omega). \tag{1}$$

The generalization of the double correlation problem to the present case, where we average over directions of intermediate radiation, is very simple. Due to the orthogonality of the spherical harmonics (1), the only finite terms in the angular correlation are those connecting the same substates of the intermediate system ($p'+F^{19*}$). After using Racah's summation rule three times, we obtain for the angular correlation function (cf. ref. [6]))

$$\begin{aligned} W(t) = \frac{4\pi}{2I_c+1} \sum_{\substack{bb'k_1k_2j_c\\ \mu_1\mu_2 mm''}} & (-)^{S_a+I_b-j_c} Z(L_aI_bL_aI_b,\, S_ak_1) \\ & W(I_bI_cI_bI_c,\, j_ck_1) Z_1(L_dI_cL_dI_c, I_dk_2) \\ \times Y^{\mu_1}_{k_1}(\Omega_a) Y^{\mu_2 *}_{k_2}(\Omega_d) & (I_ck_1m'\mu_1|I_ck_1I_cm)(I_ck_2m'''\mu_2|I_ck_2I_cm'')(m|b) \\ & (b|m'')(m'''|b')(b'|m') \times \exp\left[i(E_b-E_{b'})t/\hbar\right]. \end{aligned} \tag{2}$$

The following notation is implied by eq. (2):

$$p+F^{19} \rightarrow Ne^{20} \rightarrow p'+F^{19*} \rightarrow p'+\gamma+F^{19}$$

$$S_a+L_a \rightarrow I_b \rightarrow j_c + \underset{(=I)}{I_c} \rightarrow j_c+L_d+I_d,$$

where $S$ represents channel spin and $L$ orbital angular momentum of a particle, or multipole order of radiation, wherever appropriate. The coefficients $Z$, etc. follow customary definitions [18]), and the $(I_c k_i m' \mu_i | I_c k_i I_c m)$ are Clebsch-Gordan coefficients; $\Omega_a$ and $\Omega_d$ represent directions of entry and exit of incident proton and gamma ray. If the energy levels $E_b$, $E_{b'}$ are degenerate, (2) may be summed to a familiar form [18]) useful in calculating the undisturbed distribution. If we are to include the effect of electron shells, the transformation coefficients $(m|b)$ etc. in (2) must be replaced by the quantities $(m\mathrm{n}|\mathrm{b})(\mathrm{b}|m''\mathrm{p})(m'''\mathrm{p}|\mathrm{b}')(\mathrm{b}'|m'\mathrm{n})$ summed over the electron states n and p. A simple form is obtained for $W(t)$ if we set $\Omega_a$ along the axis $-OZ$, so that $\mu_1 = \mu_2 = 0$, $k_1 = k_2 = k$:

$$W(t) = \sum_k G_k(t) B_k P_k(\cos\theta), \tag{3}$$

where

$$G_k(t) = \sum_{mm''} (Ikm0|IkIm)(Ikm''0|IkIm'') S_{mm''}(t) \tag{3a}$$

and, following Goertzel [1]), we have put

$$S_{mm''}(t) = \sum_{\mathrm{bb'np}} \frac{(m\mathrm{n}|b)(b|m''\mathrm{p})(m''\mathrm{p}|b')(b'|m\mathrm{n})}{(2J+1)\exp[-i(E_b-E_{b'})t/\hbar]}, \tag{3b}$$

where $J$ is the electron spin; the $B_k$ are constants which define the undisturbed angular distribution.

Eq. (3) reduce to Alder's formula for isotropic H.F.S., on ignoring terms with $\mathrm{b} \neq \mathrm{b}'$. One can show that in this case the hard core for a number of independent electrons is the product of the hard cores for the single electrons. We shall generalize this statement below.

### (b) SPECIAL CASES

The special form of (2) required for the example of Section 3 is well known. In order to obtain the average distribution function $W(\theta)$ one averages $W(t)$ over all possible nuclear lifetimes. The distribution may be obtained in a simple azimuthal form, and there results the formula

$$\frac{W(0^0)-W(100^0)}{W(0^0)+W(100^0)} = a\,\frac{0.97-0.17x}{1+x^2} \tag{4}$$

with neglect of $(a/(1+x^2))$ $(0.06+0.34x)$ as compared to unity. Here $a = A/(2+A)$, $x = 2g_I \mu_0 H\tau/\hbar$, where $\mu_0$ is the nuclear magneton, $H$ the magnetic field and $\tau$ the nuclear mean lifetime. Eq. (4) was used in plotting the curve of fig. 4 to obtain $g_I$.

For the treatment of the problem of Section 5, it is necessary to use Eq. (3) with an explicit form for $S_{mm''}$. For the case $J = \frac{1}{2}$ we find

$$\begin{aligned} S_{m,\,m+1} &= \frac{k^2(m+\frac{1}{2})}{\{1+k^2(m+\frac{1}{2})\}^2}\,(1-\cos\omega t) \\ S_{m,\,m-1} &= \frac{k^2(m-\frac{1}{2})}{\{1+k^2(m-\frac{1}{2})\}^2}\,(1-\cos\omega t) \qquad (5) \\ S_{mm} &= 1-S_{m,\,m+1}-S_{m,\,m-1} \end{aligned}$$

while all other $S_{mm''}$ vanish, and we have written

$$\begin{aligned} k(M) &= \frac{\dfrac{\hbar\omega}{a}+M+(I+\frac{1}{2})x_J}{\{(I+\frac{1}{2})^2-M^2\}^{\frac{1}{2}}} \\ \frac{\hbar\omega}{a} &= (I+\tfrac{1}{2})\left(1+\frac{2M}{I+\frac{1}{2}}x_J+x_J^2\right)^{\frac{1}{2}}, \qquad (5a) \\ x_J &= \frac{-\mu_J}{J}\,\frac{H}{(I+\frac{1}{2})a} \qquad \left(g_I\mu_0 \ll -\frac{\mu_J}{J}\right), \end{aligned}$$

in which $\mu_J$ is the electronic magnetic moment and $a$ the H.F.S. constant in erg. We obtain $W(\theta)$ by averaging $S_{mm''}(t)$ over the time $T$ during which an electron remained in the atom. Putting $\omega\tau \gg 1$ and expanding in powers of $T/\tau$ the time-dependent parts of (5) lead to an integral

$$\frac{[1-(T/\tau)]\sin\omega T}{\omega T},$$

which becomes small as $\omega T$ increases. If $\omega T \geqq \pi/2$, time-dependent terms in (5) may be neglected and a "hard core" defined similarly to the case with $H = 0$. This has already been discussed in connection with table 1. Using only the constant terms of (5) it has been verified that the hard core for two independent electrons is the square of that for a single electron. It has also been verified numerically that this simplifying theorem holds for two electrons for the case $J = \frac{3}{2}$. No simple analytical formula similar to (5) can be derived in this case. We shall assume that the hard core for any number of electrons is the product of those for individual electrons taken separately. The proof for $x = 0$ and $x = \infty$ is very easy. A general proof of this doubtless depends on some form of generalised Racah summation.

Since the present problem deals with the value of $\{G_2(x_J)\}^n$ where $n = 150$, it turns out that $x_J$ is large. A perturbation calculation, accurate to order $x_J^{-2}$, has been performed and for $J = \frac{1}{2}$ or $\frac{3}{2}$ yields the simple form

$$G_2(x_J) = 1 - \frac{2J(J+1)}{9x_J^2}\,. \tag{6}$$

The recoiling nuclei must be assumed to reside in $^2P_{\frac{3}{2},\frac{1}{2}}$ states of the $F^{19}$ atom, in which the electron spins are weighted statistically, as transitions from the $P_{\frac{1}{2}}$ to the ground $P_{\frac{3}{2}}$ state will be negligible during the nuclear lifetime 19)). The curve of figure 5 was therefore plotted in terms of $x_{\frac{3}{2}}$ using the assumption 8)) that $x_J$ is proportional to $(J+1)\mu_J$. We have, in fact, plotted simply

$$G_2 B_2 = 0.4\left(1 - \frac{5}{6x_{\frac{3}{2}}^2}\,2\right)^{100}\left(1 - \frac{50}{3x_{\frac{3}{2}}^2}\right)^{50}, \tag{7}$$

which is equivalent to assuming that the anisotropy is as great as possible. The largest contributions to (7) come from $x_{\frac{1}{2}}$, though we have used it to calculate the value of a for the $P_{\frac{3}{2}}$ state.

## 7. Discussion

From the calculations of Elliott and Flowers 20)), the magnetic moment of the 197 keV state is 3.3 $\mu_0$, which implies $g_I = 1.3$. This agrees exactly with the Saclay measurement 9)), done with a liquid target in which quadrupole effects were not present. In our experiment the apparent anisotropy with zero magnetic field was $A = 0.42$ for the NaF target (Section 3) compared to $A = 0.75$ for the "true" anisotropy (Section 5). On applying the necessary correction 6)) for an attenuation coefficient $G_2 = 0.6$, our previous result is modified to

$$g_I = 1.8 \pm 0.4,$$

where the increased error takes account of uncertainty in the value of $G_2$. This result is close to the Schmidt limit for a $d_{\frac{5}{2}}$ particle, namely $g_I = 1.92$.

Anisotropies observed in the present experiments seem higher than those measured at Cal. Tech. Christy [7]) has shown that the distributions of inelastic protons and gamma-rays are related. Setting

$$W\binom{\mathrm{p}}{\gamma} = 1+B_2\binom{\mathrm{p}}{\gamma} P_2(\cos\theta)$$

one can show that

$$\frac{B_2(\mathrm{p})}{B_2(\gamma)} -= \frac{35T^2+14\sqrt{14}T}{10T^2+28},$$

where $0 < T < \infty$. Using an experimental determination of $B_2(\mathrm{p})$, Christy has shown that the "true" value of $A$ is 0.28, which is consistent with the measurement of Barnes [21]). Because of the large background in the present experiments (cf. fig. 2) we cannot rule out the possibility of systematic errors. To first order the results obtained will be unaffected by such errors: for example, the value $G_2 = 0.6$ is the same as that estimated by Christy for $\mathrm{CaF_2}$ targets at three different resonances.

In the measurement of the hyperfine structure, there may be a systematic error in the estimate of the value of $n$. It is interesting to notice that with our measured value of $g_I$, applying the theory of H.F.S. in halogen atoms [22]) the value $a = 0.18\times10^{-18}$ erg is obtained. The excellent agreement between this and the measured value $a = 0.22\times10^{-18}$ erg is perhaps somewhat fortuitous.

I wish to thank Professor E. W. Titterton for his encouragement and interest in this work. The theory in Section 6 was derived with much willing help from Dr. F. C. Barker. I am grateful to Messrs. N. F. Bowkett and W. H. Owen for technical assistance and construction of apparatus.

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*Nuclear Physics* **2** (1956/57) 254—258; *North-Holland Publishing Co., Amsterdam*

# LIFETIME MEASUREMENT BY TIME TO PULSE HEIGHT CONVERSION, APPLIED TO THE 517 keV LEVEL OF $^{85}$Rb

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Received 30 July 1956

**Abstract:** A simple method based on time to pulse height conversion, which can be used for time delay measurements in the micro-second region, has been developed. Using this method the lifetime of the $g_{9/2}$ level of $^{85}$Rb occurring in the $^{85}$Kr decay proved to be $(9.0 \pm 0.4) \times 10^{-7}$ sec, which is in good agreement with earlier measurements, and proves the identity of the $\gamma$-transitions of about 515 keV occurring in the decay of $^{85}$Kr and $^{85}$Sr.

## 1. Introduction

The energy levels of $^{85}$Rb (fig. 1) have been measured both in the decay from $^{85}$Kr$^{m}$ and $^{85}$Kr and from $^{85}$Sr$^{m}$ and $^{85}$Sr.

In the decay by K-capture of the $^{85}$Sr nucleus Sunyar *et al.* $^{1}$) measured a lifetime $\tau = (9 \pm 2) \, . \, 10^{-7}$ sec of the 513 keV $^{85}$Rb level by the delayed coincidence method. The stable ground state of $^{85}$Rb has been measured directly $^{2}$) and proved to be $f_{5/2}$, whereas from the internal conversion coefficient $^{1, 3}$) of the 513 keV $\gamma$-decay an M2 or an E3 transition could be derived; but the measured lifetime of the 513 keV level excludes an E3 transition. So this level has been interpreted as a $g_{9/2}$ level.

The decay of $^{85}$Kr to $^{85}$Rb has been investigated several times$^{4, 5, 6, 7}$), but no lifetime measurement of the 517 keV level $^{7}$) has been reported up to now. Since it would be of some importance to confirm more certainly the identity of both $\gamma$-transitions of energies of 517 keV occurring in the $^{85}$Kr-decay $^{7}$) and 513 keV occurring in the $^{85}$Sr-decay $^{1}$) respectively, we measured the lifetime of the 517 keV excited state fed by $\beta$-decay from $^{85}$Kr.

A simple method which can be used more generally for time-delay measurements in the micro-second region is developed for this purpose.

## 2. Method and Experiment

In our experiment we measured the lifetime of the isomeric 517 keV level of $^{85}$Rb occurring in the $^{85}$Kr (10.5 y) decay (fig. 1).

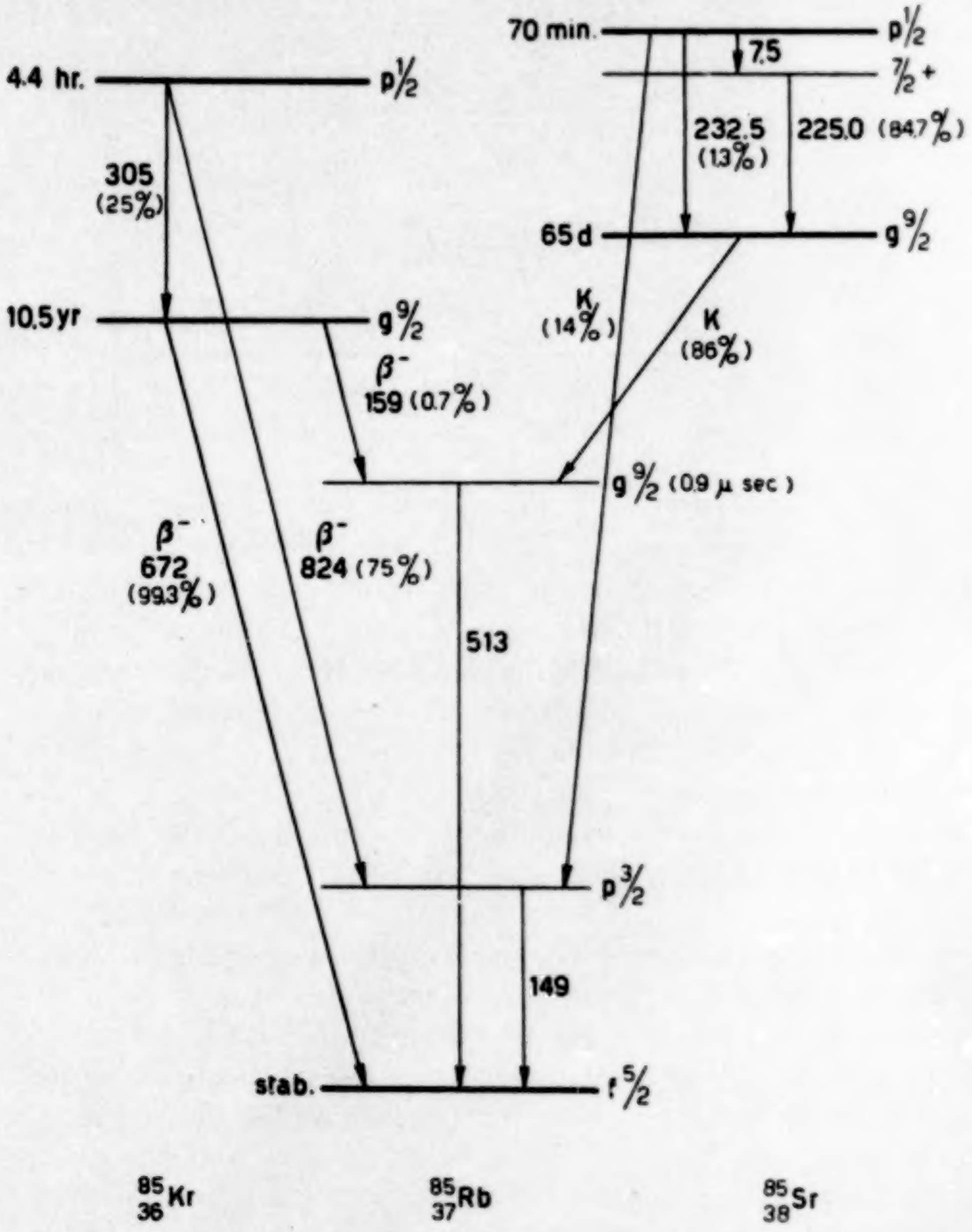
70 min.
p½
7.5
7/2 +
4.4 hr.
p½
232.5
(1.3%)
225.0 (84.7%)
305
(25%)
65 d
g 9/2
10.5 yr
g 9/2
K
(14%)
K
(86%)
β⁻
159 (0.7%)
g 9/2 (0.9 μ sec)
β⁻
672
(99.3%)
β⁻
824 (75%)
513
p 3/2
149
stab.
f 5/2
85 36 Kr
85 37 Rb
85 38 Sr

Fig. 1. Level scheme of $^{85}_{37}$Rb. Spin and parity assignments are given. The energies of the $\beta$- and $\gamma$-transitions are given in keV. Relative intensities in both decays are added between brackets to the energies.

For this purpose a delayed coincidence method with time to pulse height conversion was realized as follows (fig. 2):

Pulses from the $\beta$-sensitive channel 1 accepting the 159 keV $\beta$-transition leading to the isomeric state trigger a linear sawtooth.

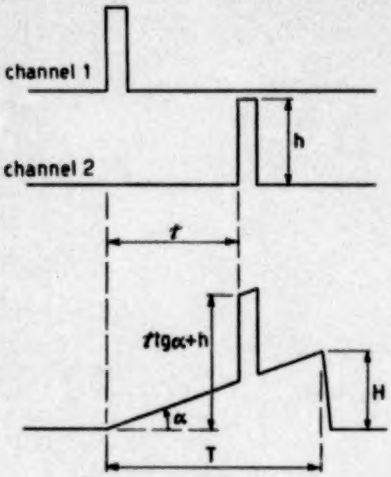

Fig. 2. Principle of thé time to pulse height conversion method.

517 keV gammas deexciting this isomeric state cause short pulses from the gamma-sensitive channel 2, which are superimposed on the sawtooth voltage. Now the height of this sum pulse is proportional to the delay of the 517 keV $\gamma$-transition and can be measured by means of a differential discriminator. It is usually necessary to make $h$ somewhat larger than $H$: in our experiment $h$ was 27 Volt and $H$ was 20 Volt.

As the lifetime of the isomeric level is of the order of 1 $\mu$sec we chose $T = 6$ $\mu$sec, whereas the short pulse formed by an EFP 60 secondary emission valve has a duration of 0.3 $\mu$sec.

The $\beta$-particles are detected with a thin stilbene scintillation crystal and EMI 6260 multiplier; this crystal is contained in a small perspex vessel which contains also $\approx 0.2$ $\mu$C $^{85}$Kr gas. The gammas are detected with a NaJ(Tl) scintillation counter.

A very low activity is required to reduce chance coincidences caused by the high intensity of the $\beta$-branch leading directly to the ground state of $^{85}$Rb.

Since, however, it is possible with this method to use a conventional multi-channel differential discriminator the time of measurement can be made relatively short.

In our case we had to use a one-channel differential discriminator which, however, was completely automatized. Two series of each about 50 runs (see fig. 3) were made, the total measuring time being almost two weeks.

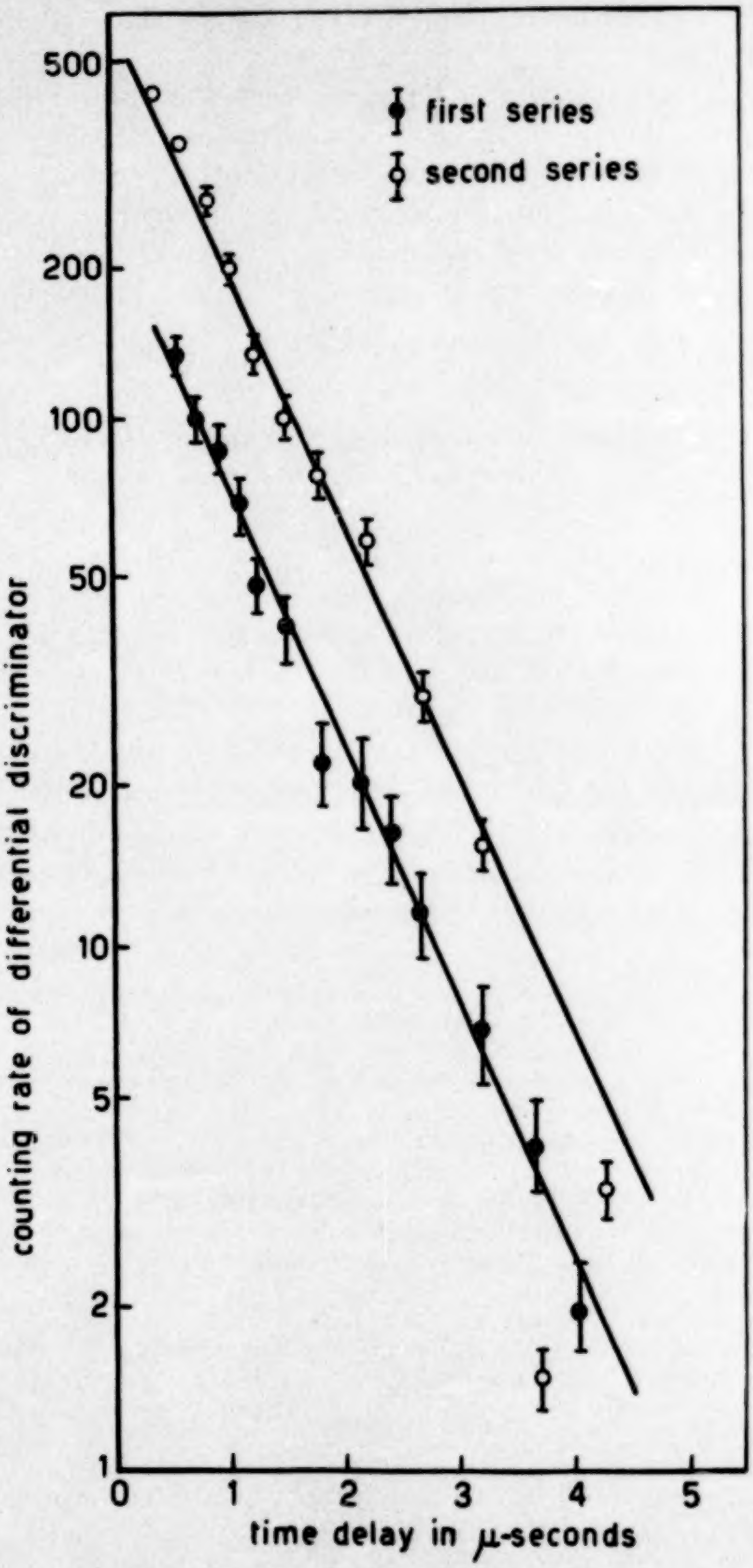

Fig. 3. One-channel differential discriminator counting-rate versus channel level expressed in time delay.

These measurements yielded decay curves that have to be corrected for

a) a slight non-linearity of the rising edge of the sawtooth pulse;

b) the contribution of the background pulses.

Having made these corrections we obtained the curves of fig. 3, in which the differential discriminator counting-rate is plotted versus the channel level translated into time delay.

The points derived from the result of the first series give

$\tau = (9.0^{+0.4}_{-1.3}) \cdot 10^{-7}$ sec, while the second series gives

$\tau = (8.7^{+0.7}_{-0.7}) \cdot 10^{-7}$ sec.

The combined measurements yield a lifetime $\tau = (9.0 \pm 0.4) \cdot 10^{-7}$ sec, which is in good agreement with the measurement of Sunyar *et al.* $^{1}$).

This confirms the identity of both $\gamma$-transitions of 517 keV in the decay of $^{85}$Kr and $^{85}$Sr respectively.

The theoretical lifetime, derived from Moszkowski's formula$^{8}$) gives a value which is 100 times smaller than the measured one.

The author is much indebted to Professor H. Brinkman for his stimulating support and to Dr. H. de Waard for drawing his attention to the problem and for the many discussions and helpful suggestions during the course of this work. Thanks are further due to Professor Hl. de Vries for kindly putting the $^{85}$Kr-gas at his disposal.

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*Nuclear Physics* **2** (1956/57) 259–266; *North-Holland Publishing Co., Amsterdam*

# CONNECTION BETWEEN SCATTERING AND MULTIPLE PRODUCTION OF PARTICLES

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Received 17 July 1956

**Abstract:** According to Fermi's theory, multiple production of particles in high energy collisions should be determined by the statistical weight of the respective state. In the present investigation the statistical weight is calculated with account of mutual interaction of pairs of particles and it is shown that in this case the scattering phase of the interacting particles is involved as a characteristic of the interaction. In resonance scattering the effect of the interaction reduces to the appearance of an intermediate "isobaric" state which should be included in the statistical theory.

## 1. Introduction

As is well known, the statistical theory of multiple production proposed by Fermi [1]) does not agree with experiment. In neutron-proton collisions at energies of the order of 2 GeV a larger number of meson pairs is produced than is predicted by the theory [2]). This discrepancy may be as high as 20 times. In ref. [3]) it was proposed to modify Fermi's theory proceeding from the following considerations.

Experiments on scattering of $\pi$-mesons by nucleons indicate that at a kinetic energy of approximately 160 MeV in the centre of mass system the scattering cross section possesses a very pronounced maximum [4]). This scattering maximum corresponds to the P state and to an isobaric spin equal to 3/2. These experiments imply that there is a strong interaction between the $\pi$ meson and the nucleon under the indicated conditions. An interaction of this type should also be manifest in multiple production of particles. It is evident that the Fermi theory in its initial form does not take this interaction into account.

Qualitatively, the strong interaction between meson and nucleon can be described by assuming the existence of an intermediate, rapidly decaying state of the nucleon-meson system with a spin equal to 3/2 and an isobaric spin of 3/2. The concept of this so-called "isobaric" state was used in a number of investigations [5, 6]) and in particular by I. E. Tamm and his coworkers [7]). We have suggested [3]) that these isobaric states, which effectively take into account strong interaction between particles, should be included in the statistical theory of multiple production. We assumed that "quasi-particles" (isobaric states of the system) may be formed in nucleon-nucleon or nucleon-meson collisions, and that these particles possess a mass of $1.32M_0$ ($M_0$ is the nucleon mass), an isobaric spin of 3/2 and an ordinary spin of 3/2. It was also assumed that the probability of formation of such "particles" was determined by their statistical weight. The modified Fermi theory was compared with several experiments on multiple production by means of nucleon-nucleon and meson-nucleon collisions in the energy range from 1.4 GeV to 5 GeV (see ref. [3, 8]). It was found that the theory satisfactorily agrees with experiment in respect of the relative probability of processes of different multiplicity and also in respect of the energy distribution of the secondary particles.

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† The Editor has learned with deep regret that the author of this paper died on the 22nd September 1956 after a sudden and brief illness.

In this paper we wish to present some general theoretical ideas concerning the connection between resonance scattering and multiple production of particles. We shall start with the following example which is due to L. D. Landau.

Let us consider the statistical partition function for a non-ideal gas. In a first approximation the gas will be considered ideal and in the second approximation only mutual interaction of pairs of particles will be taken into account [9]). As shown by Uhlenbeck and Beth [10]), the partition function which takes into account interaction of pairs of particles can be represented as follows:

$$Z = Z_1 + Z_2, \tag{1}$$

where $Z_1$ is the partition function over the discrete spectrum of two particles and $Z_2$ is that over the continuous spectrum with account of two-particle interaction:

$$Z_2 = \frac{1}{\pi}\sum_l \int_0^\infty (2l+1)\frac{\partial \eta_l}{\partial p}\, e^{-\frac{p^2}{mT}}\, dp. \tag{2}$$

Here $\eta_l(p)$ is the particle scattering phase, $p$ — the particle momentum in the centre of mass system, $l$ — the orbital momentum, $T$ — the temperature and $m$ — the particle mass. If at some value of $l$ the scattering is of a resonance nature then

$$\eta_l = \eta_l^0 + \operatorname{arc\,tg} \frac{\Gamma}{E_0 - E}, \tag{3}$$

where $\eta_l^0$ is the value of the phase at a large distance from resonance. On passing through resonance the phase changes by $\pi$. If the resonance is sufficiently sharp the expression for $Z_2$ reduces to the following:

$$Z_2 = (2l+1)\mathrm{e}^{-\frac{p_0^2}{mT}}, \tag{4}$$

where $p_0$ is the momentum value corresponding to maximum scattering. Expression (4) corresponds to an additional "discrete" positive energy state of the two-particle system.

## 2. General Theory

Returning back to the original problem, consider the collision between a $\pi$-meson and nucleon. According to the statistical theory the probability of formation of $n$ mesons during a collision is proportional to the statistical weight of a system consisting of a nucleon and $n$ mesons. Neglecting interaction between secondary particles, the statistical weight is found to be [11])

$$S_0(W, n, M_0) = \left[\frac{\Omega}{(2\pi\hbar)^3}\right]^n \frac{1}{n!} \int \prod_{i=1}^{n} \mathrm{d}\mathbf{p}_i \,.\, \mathrm{d}\mathbf{p} \,.\, \delta(W - \sum_i W_i - W_M) \\ \delta(\Sigma \mathbf{p}_i + \mathbf{p}). \tag{5}$$

Here $W$ is the energy of the system, $M_0$ the nucleon mass (in energy units), $\Omega$ the effective volume (a quantity of the order $(4\pi/3)(\hbar/\mu c)^3$, where $\mu$ is the meson mass), $\mathbf{p}_i$ — the momentum of the $i$th meson, $\mathbf{p}$ the nucleon momentum, $W_i$ — the energy of the $i$th meson and $W_M$ — the energy of the nucleon. The $\delta$ function ensures conservation of energy and momentum. Integration is carried out over momentum space of all the mesons and the nucleon.

We now compute the statistical weight with account of the interaction between nucleon and meson. Only interaction between two particles will be considered. Suppose the term corresponding to this pair of particles (nucleon and meson) is separated from expression (5). We change to the variables $\mathbf{p}_c$ and $\mathbf{p}'$ where $\mathbf{p}_c$ is the momentum of the centre of mass of the nucleon-meson system and $\mathbf{p}'$ is the momentum of the meson in the centre of mass system. The formula for the statistical weight can then be written as follows:

$$S^{(1)}(W,n,M_0)=\sum_{\mathbf{p}'}\left[\frac{\Omega}{(2\pi\hbar)^3}\right]^{n-1}\frac{1}{n!}\int\prod_{i=1}^{n-1}\mathrm{d}\mathbf{p}_i\mathrm{d}\mathbf{p}_c\,\delta\Big(W-\sum_{i=1}^{n-1}W_i-W_c-W'\Big)\,\delta\Big(\sum_{i=1}^{n-1}\mathbf{p}_i+\mathbf{p}_c\Big). \quad (6)$$

Here $W_c$ is the kinetic energy of the centre of mass of the two particles and $W'$ is their total energy in the system in which the centre of mass is at rest.

Summation over all possible states of the nucleon-meson system can be carried out in way similar to that used to calculate the partition function of a non-ideal gas [9]). As is well known, at large distances the wave function of a state with an orbital momentum $l$ and momentum $\mathbf{p}'$ has the following asymptotic form:

$$\psi \sim \mathrm{const}\,\frac{1}{r}\cdot\sin\left(\frac{p'}{\hbar}r-\frac{l\pi}{2}+\eta_l\right). \quad (7)$$

The scattering phase $\eta_l$ depends on the concrete form of the interaction between the particles. Suppose the region of variation of $r$ is restricted by some large but finite radius $R_0$. The momentum can then take the values defined by the condition

$$\frac{p'}{\hbar}R_0-\frac{l\pi}{2}+\eta_l=s\pi \quad (8)$$

where $s$ are integers. If $R_0$ is sufficiently large, $s$ can be considered to vary continuously and in expression (6) instead of taking the sum one may integrate:

$$S^{(1)}(W,n,M_0)=\sum_{\mathbf{p}'}S^{(1)}_{\mathbf{p}'}=\int\sum_{l}(2l+1)S^{(1)}_{p'}\frac{1}{\pi}\left(\frac{R_0}{\hbar}+\frac{\partial\eta_l}{\partial p'}\right)\mathrm{d}p'; \quad (9)$$

$S^{(1)}_{p'}$ is the expression under the summation sign in formula (6). The factor $(2l+1)$ and summation over $l$ are related to degeneracy with respect to the orbital momentum.

The first term in formula (9) yields the statistical weight without account of interaction, i.e., yields formula (5); the second term which takes into account two-particle interaction reduces to the following expression:

$$S^{(2)}=\sum_{l}\frac{2l+1}{\pi}\left[\frac{\Omega}{(2\pi\hbar)^3}\right]^{n-1}\frac{1}{n!}$$

$$\int\prod_{i=1}^{n-1}\mathrm{d}\mathbf{p}_i\mathrm{d}\mathbf{p}_c\,\delta\Big(W-\sum_{i=1}^{n-1}W_i-W_c-W'\Big)\,\delta\Big(\sum_{i=1}^{n-1}\mathbf{p}_i+\mathbf{p}_c\Big)\frac{\partial\eta_l}{\partial W'}\,\mathrm{d}W'. \quad (10)$$

We have changed here from the variable $p'$ to the variable $W'$ ($W'$ is the total energy in the centre of mass system) which is uniquely related to $p'$. In order to take into account the presence of $n$ mesons expression (10) should be multiplied by $n$. The expression for the statistical weight with account of interaction between the nucleon-meson pair may then be written as follows:

$$S(W, n, M_0) = S_0(W, n, M_0) + \Sigma \frac{2l+1}{\pi} \int S_0(W, n-1, W') \frac{\partial \eta_l}{\partial W'} \mathrm{d}W'. \tag{11}$$

Here the expression for $S_0(W, n, M_0)$ is given by formula (5); $S_0(W, n-1, W')$ is the statistical weight for $n$ non-interacting particles, that is, for $n-1$ mesons and one heavy "particle" whose mass is $W'$.

Suppose that at some value of $l$ scattering is of a resonance nature. If the resonance is sufficiently sharp the function $S_0(W, n-1, W')$ can be taken out from under the integral sign with its value at the point $W' = W_0$ corresponding to maximum scattering, and $\int (\partial \eta_l / \partial W') \mathrm{d}W' = \pi$. Retaining in the second part of formula (11) only the term involving resonance scattering, we get

$$S(W, n, M_0) = S_0(W, n, M_0) + (2l+1) S_0(W, n-1, W_0), \tag{12}$$

where $W_0 = M + \mu + E_0$, and $E_0$ is the kinetic energy of the nucleon and meson at resonance. On calculating the statistical weights, conservation of charge and isobaric spin should also be taken into account but this point will not be discussed here. Thus, the second term in formula (12) which accounts for the nucleon-meson interaction is exactly similar to the expression for a system of $n-1$ mesons and a nucleon in the "isobaric" state. Moreover, expression (12) in general yields the statistical weight for formation of a nucleon and $n$ mesons. It should be noted that in the general case it is not clear if two-particle interaction is sufficient to account for the interaction as a whole. However, this approximation seems to work in the case of very strong two-particle interaction involving resonance scattering.

If the collision of two nucleons is considered simultaneous interaction of the two nucleons with mesons should also be taken into account. In addition to expression (12) we obtain the following term:

$$S_{\mathrm{NN}} = \sum_{l'} \sum_{l} \frac{2l'+1}{\pi} \frac{2l+1}{\pi}$$

$$\iint S_0(W, n-2, W', W'') \frac{\partial \eta_l}{\partial W'} \frac{\partial \eta_{l'}}{\partial W''} \mathrm{d}W' \mathrm{d}W''. \quad (13)$$

Here $S_0(W, n-2, W', W'')$ is the statistical weight for $n$ non-interacting particles — for $n-2$ mesons and for two "heavy particles" whose masses are $W'$ and $W''$. If at some value of $l$ the scattering is of a resonance nature, then the equation (13) will reduce to the following expression:

$$S_{\mathrm{NN}} = (2l+1)^2 S_0(W, n-2, W_0, W_0). \quad (14)$$

The form of this expression corresponds to the statistical weight of $(n-2)$ mesons and two nucleons in the "isobaric" state. If resonance $\pi-\pi$ meson scattering is established (and there seem to be some indications of this; cf. ref. [12])) then in calculating the statistical weights this type of $\pi$ meson interaction should be taken into account in a similar manner.

## 3. Example

Let us consider the following example. Suppose that the statistical weight corresponding to the formation of a system consisting of a nucleon and two mesons is to be computed. This statistical weight is given by the formula

$$S(W, 2, M_0) = S_0(W, 2, M_0) + \sum_{l} \frac{2l+1}{\pi} \int S_0(W, 1, W') \frac{\partial \eta_l}{\partial W'} \mathrm{d}W'. \quad (15)$$

The formula for $S_0(W, 1, W')$ has a simple analytical form (see e.g. ref. [3]):

$$S_0(W, 1, W') = C(W^2 - \mu^2 - W'^2) \left[ 1 - \frac{(W'^2 - \mu^2)^2}{W^4} \right]. \quad (16)$$

Here $C$ is a coefficient which is independent of $W'$. If $(W'/W)^4 \ll 1$ the term in brackets will equal 1. We shall assume that this condition is satisfied. As a matter of fact, $W' = 1.23$ GeV and it is sufficient for $W$ to equal 2 GeV in order that the ratio $(W'/W)^4$ be of the order of 12 %.

Suppose now that at some value of $l$ there is a resonance in the scattering and that the phase change is given by formula (3). The quantity $\Gamma$ will be considered to be independent of energy †. Then

$$\frac{\partial \eta_l}{\partial W'} = \frac{\partial \eta_l}{\partial E_1} = \frac{\Gamma}{\Gamma^2 + (E_0 - E_1)^2}, \tag{17}$$

where $E_1 = W' - M_0 - \mu$. The second term in expression (16) may now be written as follows:

$$S_2 = C_1 \cdot \frac{1}{\pi} \int_0^{E_{\max}} [W^2 - \mu^2 - (M_1 + E_1)^2] \frac{\Gamma \mathrm{d} E_1}{\Gamma^2 + (E_0 - E_1)^2}, \tag{18}$$

where $C_1$ is a constant, $M_1 = M_0 + \mu$; $E_{\max} = \sqrt{W^2 - \mu^2} - M_1$. The integration can easily be carried out by changing to the variable $\varepsilon = E_1 - E_0$ and we then obtain

$$\begin{aligned} S_2 = C_1 \cdot \frac{1}{\pi} \Big[ & (W^2 - \mu^2 - M_2^2) \left( \operatorname{arc\,tg} \frac{\sqrt{W^2 - \mu^2} - M_2}{\Gamma} - \operatorname{arc\,tg} \left( -\frac{E_0}{\Gamma} \right) \right) \\ & - \Gamma M_2 \log \frac{\Gamma^2 + (\sqrt{W^2 - \mu^2} - M_2)^2}{\Gamma^2 + E_0^2} - \Gamma (\sqrt{W^2 - \mu^2} - M_2) \\ & + \Gamma^2 \left( \operatorname{arc\,tg} \frac{\sqrt{W^2 - \mu^2} - M^2}{\Gamma} - \operatorname{arc\,tg} \left( -\frac{E_0}{\Gamma} \right) \right) \Big]. \end{aligned} \tag{19}$$

Here $M_2 = M_0 + \mu + E_0$ is the nucleon mass in the "isobaric" state.

It is not difficult to see that for $\Gamma$ tending to zero

$$S_2 = C_1 (W^2 - \mu^2 - M_2^2). \tag{20}$$

Expression (20) corresponds to the statistical weight of a system of two particles — one with a mass $\mu$ (meson) and the other with a mass $M_2$ ("isobaric" nuclear state). Suppose that $\Gamma = \frac{1}{2} E_0$ and $E_0 = 160$ MeV. These values are close to those which are obtained from experiments on scattering of $\pi$ mesons by nucleons (see ref. [13]). The value of $\Gamma$ here is taken at the point $E_0$. We shall estimate the magnitude of expression (19) for $W = 1.9$ GeV. In the laboratory system this corresponds to an incident $\pi$ meson with an energy equal to 1.37 GeV. We then obtain that for $\Gamma = \frac{1}{2} E_0$, $S_2$ is 0.77 times the value of $S_2$ at $\Gamma = 0$. In the given case taking account of the level width yields a correction which apparently can be neglected in view of the approximate nature of the theory as a whole. At high energies the correction will be even smaller.

Kovacs [14]) has also made an attempt to take into account nucleon-meson interaction within the framework of statistical theory. However, in this work along with statistical concepts matrix elements were also introduced and the latter can be calculated only for concrete versions of the meson theory. The calculations are very complicated. In our opinion this combination of statistical theory (which is of a phenomenological nature) and calculations based on certain doubtful versions of meson theory is not a consistent approach to the problem at hand.

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† It can be shown that account of the dependence of $\Gamma$ on energy modifies the final results only slightly.

In conclusion I take the opportunity to thank L. D. Landau for discussion of the problems considered in this paper.

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*Nuclear Physics* **2** (1956/57), 267—276; *North-Holland Publishing Co., Amsterdam*

# LADUNGSKONJUGATION UND VERWANDTE TRANSFORMATIONEN IM NUKLEONENFELD

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Received 11 June 1956

**Abstract:** The charge conjugation and related transformations are discussed in the one-particle theory and the quantized field theory of the nucleons as well as the question of the isobaric spin of antinucleons. The components of the isobaric spin of antinucleons can be assigned without contradiction only within the framework of the quantized field theory.

## 1. Ladungskonjugation in der Einteilchentheorie

J. V. Lepore behandelt in seiner vor kurzem erschienenen Arbeit [1]) die in Betracht kommenden Definitionen der Ladungskonjugation im Nukleonenfeld. Sein Ziel war es, die Komponenten des isobarischen Spins für die Antinukleonen festzustellen. Es scheint nicht überflüssig zu sein, zu den Problemen, die in der erwähnten Arbeit aufgegriffen werden, einige neue Gesichtspunkte hinzuzufügen, die zum Verständnis des Wesens der behandelten Transformationen beitragen können.

Lepore's Ausgangspunkt bildet die Bemerkung, dass im Falle der achtkomponentigen Diracschen Gleichung des Nukleons

$$\left[\left(\partial_i - \frac{ie}{\hbar c} A_i \frac{1+\tau_3}{2}\right)\gamma_i + \varkappa\right]\psi = 0 \tag{1}$$

die Matrix $C$ der Ladungskonjugation durch die Bedingungen

$$C^{-1}\gamma_i C = -\gamma_i^{\mathrm{T}}, \quad C^{+} = C^{-1} \tag{2}$$

keineswegs eindeutig bestimmt wird. Ist nämlich $C'$ die Schwingersche Vierermatrix der Ladungskonjugation, die bis auf einen Faktor vom Absolutwerte eins eindeutig definiert ist, so wird (2) durch jede Achtermatrix von der Form

$$C = T \times C' \tag{3}$$

befriedigt, wo $T$ eine beliebige unitäre Matrix vom zweiten Rang ist ($\times$ ist das Zeichen des direkten Produktes). Die Wellenfunktion

$$\varphi = C\bar{\psi}^{\mathrm{T}} \qquad (\bar{\psi} = \psi^{+}\gamma_4) \tag{4}$$

befriedigt dann die Gleichung

$$\left[\left(\partial_\iota + \frac{ie}{\hbar c} A_\iota \frac{1+\tau_3'}{2}\right)\gamma_\iota + \varkappa\right]\varphi = 0. \tag{5}$$

Hier ist $\tau_3' = T\tau_3 T^{-1}$; $^{\mathrm{T}}$ ist das Zeichen der transponierten, $^{+}$ ist das Zeichen der komplex konjugierten und transponierten Wellenfunktion. Wenn wir neben (2) noch die Forderung

$$C^{\mathrm{T}} = -C \tag{2a}$$

stellen, so definiert (4) eine symmetrische Relation zwischen $\psi$ und $\varphi$; wir haben dann

$$\psi = C\bar{\varphi}^{\mathrm{T}}. \tag{4a}$$

Aus (2a) folgt wegen $C'^{\mathrm{T}} = -C'$ und wegen (3)

$$T^{\mathrm{T}} = T. \tag{2b}$$

Drücken wir $T$ mittels der Matrizen des vollständigen Paulischen Matrizen-Systems vom zweiten Rang

$$\tau_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

aus, dann können wir, die Unitarität und Gl. (2b) berücksichtigend, schreiben:

$$T = \sin\vartheta\,(\cos\varphi\cdot\tau_0 + i\sin\varphi\cdot\tau_1) + i\cos\vartheta\cdot\tau_3, \tag{6}$$

mit willkürlichen Winkelvariablen $\vartheta$, $\varphi$.

Wenn wir aber die Symmetrie der Relation (4) in $\psi$ und $\varphi$ nur bis auf das Vorzeichen bewahren wollen, so kann anstatt (4a)

$$\psi = -C\bar{\varphi}^{\mathrm{T}} \tag{4b}$$

gefordert werden. Dies wird erfüllt, wenn

$$C^{\mathrm{T}} = C, \; T^{\mathrm{T}} = -T, \quad \text{d.h. } T = \tau_2 \tag{2c}$$

ist.

Untersuchen wir nun näher die Transformationen[†]

$$\varphi_\alpha = C_\alpha\bar{\psi}^{\mathrm{T}}, \; C_\alpha = \tau_\alpha \times C' \qquad (\alpha = 0, 1, 2). \tag{4c}$$

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[†] Die Transformationen $\alpha = 0, 1$ sind Spezialfälle von (3), (6), die Transformation $\alpha = 2$ ist identisch mit (3), (2c).

Die Funktionen (4c) befriedigen die folgenden Gleichungen:

$$\begin{aligned} &\left[\left(\partial_i+\frac{ie}{\hbar c}A_i\frac{1+\tau_3}{2}\right)\gamma_i+\varkappa\right]\varphi_0=0, \\ &\left[\left(\partial_i+\frac{ie}{\hbar c}A_i\frac{1-\tau_3}{2}\right)\gamma_i+\varkappa\right]\varphi_\alpha=0 \qquad (\alpha=1,2). \end{aligned} \tag{5a}$$

Man würde hieraus folgern, dass wenn $\varphi_0$ als die Wellenfunktion des Antinukleons betrachtet wird, folgt [1])

$$\bar{\mathrm{p}} \rightarrow (\tau_3=1), \qquad \bar{\mathrm{n}} \rightarrow (\tau_3=-1),$$

wenn man aber für die Beschreibung des Antinukleons $\varphi_\alpha$ $(\alpha=1, 2)$ wählt, so ist

$$\bar{\mathrm{p}} \rightarrow (\tau_3=-1), \qquad \bar{\mathrm{n}} \rightarrow (\tau_3=1).$$

Diesem Schluss liegt die Annahme zugrunde, dass (5a) als die Zustandsgleichung eines Antinukleons betrachtet werden kann. Es soll aber daran erinnert werden, dass die Ladungskonjugation schon in der Diracschen Einteilchentheorie des Negatons und des Positons nicht widerspruchsfrei durchgeführt werden kann [2]). (Diese Frage wird im Anhang eingehender diskutiert.) Die Komponenten des isobarischen Spins sollen darum auf Grund der quantisierten Feldtheorie den Antinukleonen zugeschrieben werden.

## 2. Ladungskonjugation in der quantisierten Feldtheorie

Der Operator des Nukleonenfeldes wird aus dem Operator des Protonenfeldes $\chi$ und aus dem Operator des Neutronenfeldes $\omega$ folgendermassen zusammengesetzt:

$$\psi=\begin{pmatrix}\chi\\ \omega\end{pmatrix};$$

$\chi$ und $\omega$ können im kräftefreien Falle mittels der Emissions- und Absorptionsoperatoren folgendermassen dargestellt werden:

$$\begin{aligned} \chi&=\sum_n\left(a_n u_n^{(+)}+b_n^+ u_n^{(-)}\right), \\ \omega&=\sum_n\left(c_n u_n^{(n)}+d_n^+ u_n^{(-)}\right). \end{aligned} \tag{7}$$

Die Bedeutung der Operatoren $a$, $b$, $c$, $d$ ist die folgende:

| | p | $\overline{\mathrm{p}}$ | n | $\overline{\mathrm{n}}$ |
|---|---|---|---|---|
| Emissionsoperator | $a^+$ | $b^+$ | $c^+$ | $d^+$ |
| Absorptionsoperator | $a$ | $b$ | $c$ | $d$ |

(8)

Betrachten wir die Komponente $T_3$ des isobarischen Spins:

$$T_3 = \tfrac{1}{2}\int \psi^+\tau_3\psi \, \mathrm{dV} = \tfrac{1}{2}\int (\chi^+\chi - \omega^+\omega) \, \mathrm{dV}$$
$$= \tfrac{1}{2}\sum_n (a_n^+ a_n - b_n^+ b_n - c_n^+ c_n + d_n^+ d_n). \tag{9}$$

Aus (9) ist ersichtlich, dass $T_3$ auf die folgende Weise auf die Einteilchenzustände $\mathrm{P} = a^+\Omega_0$, $\overline{\mathrm{P}} = b^+\Omega_0$, $\mathrm{N} = c^+\Omega_0$, $\overline{\mathrm{N}} = d^+\Omega_0$ ($\Omega_0$ = Zustandsvektor des Vakuums) wirkt:

$$T_3\mathrm{P} = \tfrac{1}{2}\mathrm{P}, \qquad T_3\mathrm{N} = -\tfrac{1}{2}\mathrm{N},$$
$$T_3\overline{\mathrm{P}} = -\tfrac{1}{2}\overline{\mathrm{P}}, \qquad T_3\overline{\mathrm{N}} = \tfrac{1}{2}\overline{\mathrm{N}}.$$

Es ist ersichtlich, dass die Eigenwerte von $T_3$ sich eindeutig aus der quantisierten Feldtheorie ergeben, und keineswegs davon abhängen, „wie die Wellenfunktion der Antinukleonen definiert wird”:

| | p | $\overline{\mathrm{p}}$ | n | $\overline{\mathrm{n}}$ |
|---|---|---|---|---|
| $T_3$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ |

Die Matrixform des Operators $\mathbf{T}$ im System der Einteilchenzustände P, N, $\overline{\mathrm{P}}$, $\overline{\mathrm{N}}$ lautet folgendermassen:

$$\mathbf{T} = \tfrac{1}{2}\begin{pmatrix} \boldsymbol{\tau} & 0 \\ 0 & -\boldsymbol{\tau} \end{pmatrix}.$$

## 3. Die Bedeutung der Transformationen $C_0$, $C_1$, $C_2$

Nun soll die physikalische Bedeutung der Transformationen des Feldoperators $\psi$

$$\psi_\alpha = C_\alpha \overline{\psi}^{\mathrm{T}} \qquad (\alpha = 0, 1, 2) \tag{10}$$

erörtert werden. (Die Matrizen $C_\alpha$ sind durch (4c) definiert.) Gl. (7) kann in zusammenfassender Schreibweise folgendermassen dargestellt werden:

$$\psi = \sum_n (a_n f_n^{(+)} + b_n^+ f_n^{(-)} + c_n g_n^{(+)} + d_n^+ g_n^{(-)}). \tag{11}$$

Hier ist

$$f_n^{(\pm)} = \begin{pmatrix} u_n^{(\pm)} \\ 0 \end{pmatrix}, \quad g_n^{(\pm)} = \begin{pmatrix} 0 \\ u_n^{(\pm)} \end{pmatrix}. \tag{12}$$

Es ist leicht einzusehen, dass

$$\tau_1 f = g, \qquad \tau_2 f = ig, \qquad \tau_3 f = f,$$
$$\tau_1 g = f, \qquad \tau_2 g = -if, \qquad \tau_3 g = -g.$$

Wie bekannt, können die Eigenfunktionen $u_n$ so gewählt werden, dass eine eindeutige Zuordnung von der Form

$$u_n^{(-)} = C' \overline{u_n^{(+)}}^{\mathrm{T}}$$

zwischen den Funktionen positiver und negativer Energie besteht. Nun kann man sich leicht überzeugen, dass die Reihenentwicklungen der Operatoren (10) folgendermassen lauten:

$$\begin{aligned} \varphi_0 &= \sum_n (b_n f_n^{(+)} + a_n^+ f_n^{(-)} + d_n g_n^{(+)} + c_n^+ g_n^{(-)}), \\ \varphi_1 &= \sum_n (d_n f_n^{(+)} + c_n^+ f_n^{(-)} + b_n g_n^{(+)} + a_n^+ g_n^{(-)}), \\ \varphi_2 &= \sum_n (-i d_n f_n^{(+)} - i c_n^+ f_n^{(-)} + i b_n g_n^{(+)} + i a_n^+ g_n^{(-)}). \end{aligned} \tag{13}$$

Die Transformationen $\psi \to \varphi_\alpha$ $(\alpha = 0, 1, 2)$ lassen die Lagrange-Funktion und die Vertauschungsrelationen invariant, infolgedessen sind sie kanonisch, und können mittels gewisser Operatoren $U_\alpha$ folgendermassen dargestellt werden:

$$\varphi_\alpha = U_\alpha \psi U_\alpha^{-1} \qquad (\alpha = 0, 1, 2). \tag{14}$$

Aus (13) sieht man, dass $U_0$ die folgende Substitution der Operatoren *a*, *b*, *c*, *d* beschreibt:

$$\mathrm{p} \to \overline{\mathrm{p}}, \quad \overline{\mathrm{p}} \to \mathrm{p}, \quad \mathrm{n} \to \overline{\mathrm{n}}, \quad \overline{\mathrm{n}} \to \mathrm{n}; \tag{15}$$

$U_1$ und $U_2$ beschreiben dagegen die Substitutionen

$$\mathrm{p} \to \overline{\mathrm{n}}, \quad \overline{\mathrm{p}} \to \mathrm{n}, \quad \mathrm{n} \to \overline{\mathrm{p}}, \quad \overline{\mathrm{n}} \to \mathrm{p}. \tag{16}$$

Aus (15) und (16) geht hervor, dass der Name „Ladungskonjugation" nur von $U_0$ verdient wird, nur dessen Eigenwerte bleiben bei elektromagnetischen Wechselwirkungen erhalten. Im Falle ladungsunabhängiger Wechselwirkungen sind dagegen auch die Eigenwerte von $U_1$ und $U_2$ „gute Quantenzahlen". Die explizite Form von $U_0$, $U_1$, $U_2$ kann mittels einer von Wick [3]) vorgeschlagenen Methode aufgefunden werden:

$$\begin{aligned}
U_0 &= \prod_n (1-a_n^+a_n-b_n^+b_n+a_n^+b_n+b_n^+a_n)(1-c_n^+c_n-d_n^+d_n+c_n^+d_n+d_n^+c_n),\\
U_1 &= \prod_n (1-a_n^+a_n-d_n^+d_n+a_n^+d_n+d_n^+a_n)(1-c_n^+c_n-b_n^+b_n+c_n^+b_n+b_n^+c_n), \qquad (17)\\
U_2 &= \prod_n (1-a_n^+a_n-d_n^+d_n+ia_n^+d_n+id_n^+a_n)(1-c_n^+c_n-b_n^+b_n-ic_n^+b_n-ib_n^+c_n).
\end{aligned}$$

Betrachten wir nun die Diracsche Gleichung des Nukleonenfeldes, das mit einem symmetrischen pseudoskalaren Felde mit pseudoskalarer Kopplung in Wechselwirkung steht:

$$(\partial_i\gamma_i+ig\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\varphi}+\varkappa)\psi = 0. \qquad (18)$$

Das Potential $\boldsymbol{\varphi}$ des Mesonenfeldes muss im Falle der Transformationen (17) folgendermassen transformiert werden, wenn die Invarianz der Diracschen Gleichung gefordert wird:

$$\begin{array}{lll}
\psi \to \varphi_0, & \psi \to \varphi_1, & \psi \to \varphi_2,\\
\phi_1 \to \phi_1, & \phi_1 \to \phi_1, & \phi_1 \to -\phi_1,\\
\phi_2 \to -\phi_2, & \phi_2 \to \phi_2, & \phi_2 \to -\phi_2,\\
\phi_3 \to \phi_3, & \phi_3 \to -\phi_3, & \phi_3 \to -\phi_3.
\end{array} \qquad (19)$$

Führt man die ladungvermehrenden und -vermindernden Operatoren

$$\phi^+ = \frac{1}{\sqrt{2}}(\phi_1-i\phi_2), \qquad \phi = \frac{1}{\sqrt{2}}(\phi_1+i_1\phi_2)$$

ein, so lauten die Transformationen (19):

$$\begin{array}{lll}
\psi \to \varphi_0, & \psi \to \varphi_1, & \psi \to \varphi_2,\\
\phi \to \phi^+, & \phi \to \phi, & \phi \to -\phi,\\
\phi_3 \to \phi_3, & \phi_3 \to -\phi_3, & \phi_3 \to -\phi_3.
\end{array} \qquad (20)$$

(20) kann auch mittels der Mesonenoperatoren

| | $\pi^+$ | $\pi^0$ | $\pi^-$ |
|---|---|---|---|
| Emissionsoperator | $p_{+1}^+$ | $p_0^+$ | $p_{-1}^+$ |
| Absorptionsoperator | $p_{+1}$ | $p_0$ | $p_{-1}$ |

ausgedrückt werden:

$$\begin{array}{lll}
U_0p_{\pm1}U_0^{-1} = p_{\mp1}, & U_1p_{\pm1}U_1^{-1} = p_{\pm1}, & U_2p_{\pm1}U_2^{-1} = -p_{\pm1},\\
U_0p_0U_0^{-1} = p_0, & U_1p_0U_1^{-1} = -p_0, & U_2p_0U_2^{-1} = -p_0.
\end{array} \qquad (21)$$

Hieraus folgt, dass derjenige Teil der Operatoren $U_0$, $U_1$, $U_2$, der auf die Mesonenvariablen wirkt, von der Gestalt

$$
\begin{aligned}
U_0 &= (-1)^{\frac{1}{2}\sum\limits_n (p^+_{+1,n} p_{+1,n} + p^+_{-1,n} p_{-1,n} - p^+_{+1,n} p_{-1,n} - p^+_{-1,n} p_{+1,n})}, \\
U_1 &= (-1)^{\sum\limits_n p^+_{0,n} p_{0,n}}, \qquad (22) \\
U_2 &= (-1)^{\sum\limits_n (p^+_{+1,n} p_{+1,n} + p^+_{0,n} p_{0,n} + p^+_{-1,n} p_{-1,n})}.
\end{aligned}
$$

ist.

## 4. Vertauschungsrelationen und Auswahlregeln

Es sollen nun die Vertauschungsrelationen der Operatoren $U_0$, $U_1$, $U_2$ bzw. $Q$, $\mathbf{T}$, $N$ angegeben werden, wo $Q$ die Gesamtladung, $\mathbf{T}$ den isobarischen Spin und $N$ die Teilchenzahl (genauer: die Differenz der Zahl der Teilchen und Antiteilchen) des Nukleonenfeldes darstellen:

$$
\begin{aligned}
&[U_0, Q] = 2U_0 Q, \qquad [U_0, N] = 2U_0 N, \\
&[U_1, Q] = U_1 N, \qquad [U_1, N] = 2U_1 N, \qquad [U_2, \mathbf{T}] = 0. \\
&[U_2, Q] = U_2 N, \qquad [U_2, N] = 2U_2 N.
\end{aligned}
$$

Aus diesen Relationen folgt, dass $U_0$ nur dann mit der Ladung und der Nukleonenzahl gleichzeitig diagonalisierbar ist, wenn $N = Q = 0$ ist. Die Bedingung für die Diagonalisierbarkeit von $U_1$ und $U_2$ lautet: $N = 0$. Der Operator $U_2$ hat den speziellen Vorteil gegenüber $U_1$, dass er auch mit dem isobarischen Spin vertauschbar ist. So können mittels seiner Eigenwerte ganze Ladungsmultiplette charakterisiert werden. Dies wurde zuerst von Yang und Lee hervorgehoben [4]).

Aus (22) folgt, dass Zustände mit der Gesamtzahl $n_\pi$ der Pionen zum Eigenwerte $(-1)^{n_\pi}$ vom $U_2$ gehören; solche Zustände, in denen $n_{\pi^0}$ neutrale Pionen vorhanden sind, können mittels des Eigenwertes $(-1)^{n_{\pi^0}}$ vom $U_1$ charakterisiert werden.

Aus der Erhaltung von $(-1)^{n_{\pi^0}}$ folgt in einer sehr einfachen Weise ein Satz von Pais und Jost, nach dem eine geschlossene Nukleonenlinie mit einer ungeraden Anzahl von neutralen Mesonenknotenpunkten (mit $\tau_3$-Kopplung) auf eine verschwindende Übergangswahrscheinlichkeit führt. (Dieser Satz wurde von Pais und Jost auf Grund der Untersuchung der Streumatrix abgeleitet [5]).) Auf andere Auswahlregeln, die aus der Erhaltung von $U_0$, $U_1$, $U_2$ folgen, wurde schon von mehreren Autoren hingewiesen [3, 4, 6, 7]).

## Anhang

Es sollen hier einige Bemerkungen über die Definition der physikalischen Grössen in der Diracschen Einteilchentheorie gemacht werden. Zuerst betrachten wir den isobarischen Spin. Die Zustandsgleichung des Nukleons lautet

$$\left[\left(\partial_k - \frac{ie}{\hbar c} A_k \frac{1+\tau_3}{2}\right)\gamma_k + \varkappa\right]\psi = 0.$$

Die ladungskonjugierte Funktion $\varphi = C_0 \psi^{\mathrm{T}}$ befriedigt die Gleichung

$$\left[\left(\partial_k + \frac{ie}{\hbar c} A_k \frac{1+\tau_3}{2}\right)\gamma_k + \varkappa\right]\varphi = 0.$$

Man kann sich leicht überzeugen, dass aus

$$\tau_3\psi = \pm\psi \qquad \tau_3\varphi = \pm\varphi$$

folgt. So erhält man, dass aus

$$\int \psi^{+}(\tfrac{1}{2}\tau_3)\,\psi\,\mathrm{d}V = \pm\tfrac{1}{2} \qquad\qquad \int \varphi^{+}(\tfrac{1}{2}\tau_3)\,\varphi\,\mathrm{d}V = \pm\tfrac{1}{2}$$

folgt.

Hieraus würde man folgern, dass die Komponenten des isobarischen Spins der Antikukleonen die folgenden sind (siehe S. 269):

$$\overline{\mathrm{p}} \rightarrow (\tau_3 = 1), \qquad \overline{\mathrm{n}} \rightarrow (\tau_3 = -1).$$

Hiermit steht das Resultat, das sich auf Grund der Definition des isobarischen Spins als Drehungsoperator im Ladungsraume ergibt, im Widerspruch. Aus

$$\psi' = \mathrm{e}^{\frac{i}{2}\tau_3\alpha}\,\psi \quad \text{folgt} \quad \varphi' = \mathrm{e}^{-\frac{i}{2}\tau_3\alpha}\,\varphi.$$

Man würde auf diese Weise schliessen, dass

$$\overline{\mathrm{p}} \rightarrow (\tau_3 = -1), \qquad \overline{\mathrm{n}} \rightarrow (\tau_3 = 1).$$

Die geschilderte Situation ähnelt der, der man bei der Definition der Ladung begegnet. Wenn man die Ladungsdichte mittels der ladungskonjugierten Funktion eines Elektrons bildet, so erhält man, dass das Vorzeichen der Ladungsdichte ungeändert bleibt. Infolgedessen folgt aus

$$\int (e\psi^{+}\psi)\,\mathrm{d}V = e, \qquad \int (e\varphi^{+}\varphi)\,\mathrm{d}V = e.$$

Wenn aber die Ladung mittels des Operators der Eichtransformation definiert wird:

$$A'_k = A_k + \partial_k \lambda, \qquad \psi' = \exp\left(e\frac{i\lambda}{\hbar c}\right)\psi, \qquad \varphi' = \exp\left(-e\frac{i\lambda}{\hbar c}\right)\varphi,$$

so findet man, dass das Vorzeichen der Ladung bei einer Ladungskonjugation umgekehrt wird. Man sieht also, dass die Definition der physikalischen Grössen auf Grund der Transformationseigenschaften zu richtigen Resultaten führt, die Dichte der Ladung und des isobarischen Spins erfährt dagegen kein Vorzeichenumkehr im Gegensatz zu unserer Erwartung. Den Grund dieser Diskrepanz muss man in dem Umstand suchen, dass die Grössen $\psi^+\psi$ und $\psi^+\tau_3\psi$ in der Einteilchentheorie bzw. in der quantisierten Theorie bei einer Ladungskonjugation verschiedenartig transformiert werden. In der Einteilchentheorie gilt nämlich

$$\psi^+\psi = \varphi^+\varphi, \qquad \psi^+\tau_3\psi = \varphi^+\tau_3\varphi.$$

In der Theorie des nach dem Ausschliessungsprinzip gequantelten Feldes gilt dagegen

$$\psi^+\psi = -\varphi^+\varphi \;(+\text{konst,}), \qquad \psi^+\tau_3\psi = -\varphi^+\tau_3\varphi(+\text{konst.}).$$

Unsere Betrachtungen zeigen, dass die scheinbare Mehrdeutigkeit der Definition der Ladungskonjugation leicht beseitigt werden kann, da es unter den in Betracht kommenden Transformationen im wesentlichen nur eine gibt, die die Neutronen- und Protonenzustände nicht vermischt. Die Komponenten des isobarischen Spins der Antinukleonen können auf Grund der quantisierten Feldtheorie auf eindeutige Weise festgestellt werden. Die künstliche Unterscheidung zwischen den „wahren" und den „effektiven" Werten der Komponenten [1]) erscheint überflüssig. Die Komponenten des isobarischen Spins gehören mit der Ladung, den Komponenten des Spins und des Impulses, und der Energie zu der Klasse der Quantenzahlen, die — im Sinne der Löchertheorie — im ladungskonjugierten Zustande den entgegengesetzten Wert annehmen wie im ursprünglichem Zustand. Die Quantenzahl von Gell-Mann und Sachs (strangeness, attribute) [8, 9]) gehört auch in diese Klasse, da sie mittels $Q$, $N$ und $T_3$ ausgedrückt werden kann.

Herrn Prof. L. Rosenfeld möchte ich meinen innigsten Dank aussprechen für seine fördernde Kritik. Ich danke den Herren Prof. I. Kovács und Prof. K. F. Novobátzky die mir die Möglichkeit gaben diese Arbeit in der stimulierenden Atmosphäre des Institutes für Theoretische Physik der Roland Eötvös Universität, Budapest

ausführen zu können, sowie Herrn Dr. G. Marx für einige wertvolle Diskussionen.

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*Nuclear Physics* **2** (1956/57) 277—280; *North-Holland Publishing Co., Amsterdam*

# CONCERNING THE NUMBER OF DIFFERENT TYPES OF K-MESONS

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Received 10 August 1956

**Abstract:** The small angle scattering cross-sections of K-mesons by deuterons with and without charge exchange are estimated and their bearing on the question of the number of different types of K-mesons discussed. Possible interpretations are examined for the difference of K-meson scattering by free and bound nucleons.

## 1. K-Meson Scattering by Deuterons

As is well known, within the accuracy of the experimental results the masses and lifetimes of K-mesons decaying into 2 $\pi$-mesons ($K_{\pi 2}$ decay) or 3 mesons ($K_{\pi 3}$ decay) appear to be identical. If one assumes that the spins of these K-mesons are equal to zero (and no experimental facts have been found which testify to the contrary) the following possibilities arise.

(a) $K_{\pi 2}$ and $K_{\pi 3}$ decays are attributable to a single type of K-meson (for example, a $\tau$-meson). In this case if all the $\pi$-mesons are considered to be pseudoscalar the parity of the produced $\pi$-mesons in one of the decay schemes must differ from that of the decaying K-meson.

(b) $K_{\pi 2}$ and $K_{\pi 3}$ decays involve different K-mesons, one being scalar $\theta$, and the other pseudoscalar $\tau$. In this case identity of the masses and lifetimes would require an explanation.

At present it is not possible to exclude either of these possibilities. Further experiments are necessary to permit one to ascertain whether one or two types of K-mesons exist. In this connection it would seem to be of particular interest to study the charge exchange reaction and inelastic scattering of K-mesons on deuterons, that is,

$$K^+ + d \rightarrow K^0 + 2p, \tag{1}$$

$$K^+ + d \rightarrow K^+ + p + n, \tag{2}$$

under conditions when the scattered $K^+$-meson (or produced $K^0$

meson) is emitted in the forward direction and its momentum is approximately equal to that of the incident K⁺-meson.

The energy of the nucleons produced in reactions (1) and (2) should be of the order of several MeV.

The cross sections for reactions (1) and (2) can easily be computed in the impulse approximation, both for the case when the K-meson parity changes during the indicated reactions (this being possible if two types of K-mesons, $\theta$ and $\tau$, exist) and for the case when the K-meson parity does not change.

Consider scattering and charge exchange of K⁺-mesons on nucleons for the case when the parity changes:

$$\tau^+ + \mathrm{p} \rightarrow \theta^+ + \mathrm{p} \tag{3}$$

$$\tau^+ + \mathrm{n} \rightarrow \theta^+ + \mathrm{n} \tag{4}$$

$$\tau^+ + \mathrm{n} \rightarrow \theta^0 + \mathrm{p}. \tag{5}$$

The amplitudes for these reactions have the form $\varrho_i \boldsymbol{\sigma}\cdot\mathbf{k} \exp(i\varphi_i)$ $(i = 3, 4, 5)$where $\mathbf{k}$ is the K-meson momentum and $\varrho$ and $\varphi$ are functions of $\mathbf{k}$; the respective cross sections are $\sigma_i = \varrho_i^2 k^2$.

The cross sections for reactions (1) and (2) calculated in the impulse approximation can be expressed in terms of $\varrho_i$ and $\varphi_i$ as follows:

$$\sigma_1 = \tfrac{2}{3}\varrho_5^2 k^2,$$
$$\sigma^2 = \tfrac{1}{3}[\varrho_3^2 + \varrho_4^2 - 2\varrho_3\varrho_4 \cos(\varphi_3 - \varphi_4)]k^2.$$

Isobaric invariance yields $\varrho_3 \exp(i\varphi_3) = \varrho_4 \exp(i\varphi_4) + \varrho_5 \exp(i\varphi_5)$. Thus, if cross section $\sigma_1$ is not zero, cross section $\sigma_2$ should also differ from zero and vice versa. It can easily be seen that excluding the case when $\varrho_4 = \varrho_3$, $\varphi_4 = \varphi_3$ cross sections $\sigma_1$ and $\sigma_2$ do not vanish.

On the other hand it can be shown in a similar manner that if the K-meson parity remains constant during the reaction the cross sections for processes (1) and (2) will tend to zero as $(\Delta\mathbf{k}/2\alpha)^2$ when $\Delta\mathbf{k}$ approaches zero. Here $\Delta\mathbf{k}$ is the change in the K-meson momentum during the reaction, $\alpha = \sqrt{M\varepsilon}$, where $M$ is the nucleon mass and $\varepsilon$ is the deuteron binding energy. Thus, emission of K-mesons in reactions (1) and (2) at angles $\theta$ which are much smaller than $\theta_0 = 2\sqrt{M\varepsilon}/k = \sqrt{2M\varepsilon/M_K E}$ should signify the existence of two types of K-mesons, $\theta$ and $\tau$, which change into each other in nuclear interactions.

If reactions (1) and (2) are forbidden for small angles two possibilities arise: 1) only one type of meson (scalar or pseudoscalar) exists; 2) two types of K-mesons exist, but their mutual transformation in nuclear interactions is forbidden. (It is not excluded, of course, that the cross sections for reactions (1) and (2) may be small due to smallness of the cross section for reaction (5)).

It should be emphasized that the interpretation of the experimental results proposed here fundamentally rests on the assumption that the K-meson spin is zero. If the K-meson spin is not zero disintegration and charge exchange for K-mesons scattered into small angles may take place, even if the K-meson parity remains unchanged during the reaction.

## 2. K-Meson Scattering by Free and Bound Nucleons

It has been found experimentally that the interaction cross section of K-mesons on free protons is appreciably greater than the interaction cross section of K-mesons on nucleons bound in the nucleus [1, 2]). The cross section on hydrogen is in fact of the order of 20—28 mb, whereas the cross section per nucleon in the heavy nuclei of photographic emulsions is of the order of 6—9 mb.

It seems to us that the assumption [3]) that K-mesons predominantly interact with nucleons in the state with $T = 1$ is not the only possible explanation of this discrepancy. In this case the cross section for charge exchange and the cross section for elastic scattering of $K^+$-mesons by neutrons are equal to each other, each of them equalling 1/4 of the $K^+$-meson scattering cross section on protons. Even if it be taken into account that charge exchange in heavy nuclei is a hindered process (see below) one finds that even in this case the cross section per nuclear neutron is $\geqq 1/4$ of the cross section on a proton, a result which is difficult to reconcile with the experimental evidence.

The best agreement with experiment could also be obtained by assuming that the sum of the isobaric triplet and singlet amplitudes is small. Then $\sigma_4 \ll \sigma_5 \approx \sigma_3$, that is, the charge exchange cross section in K-meson-neutron encounters is considerably greater than the elastic scattering cross section. The charge exchange reaction in nuclei at K-meson energies $\lessapprox 50$ MeV, however, is a hindered process. The explanation of this is that on charge exchange the neutron must transform into a proton, but since all proton states in the nucleus are occupied, the produced proton must either go over onto an excited nuclear level or into the continuous spectrum. (A similar decrease in the charge exchange cross section is known for $\pi$-mesons [4]). The charge exchange cross section per nucleon in nuclei is 5 to 10 times smaller for 50 MeV $\pi$-mesons than the charge exchange cross section on free nucleons). As a result, nuclear neutrons practically do not participate in the scattering of K-mesons. It should be stressed that the scattering cross section on nuclear protons does not change in this case. A straightforward check of the assumption of a large charge exchange cross section can be made by studying large angle scattering of K-mesons in deuterium, since in this case the Pauli principle is not essential.

Another possible explanation of the anomalously small magnitude of the interaction cross section of K-mesons on nuclei is that two types of K-mesons (scalar and pseudoscalar) may exist. If the parity of the meson changes in nucleon-K-meson scattering the nucleon spin will be reversed. In nuclei this process should be hindered in virtue of the Pauli principle (similarly to the case of charge exchange). Thus if the parity exchange cross section comprises an appreciable fraction of the total K-meson-nucleon interaction cross section, the cross section on nuclei should be considerably reduced. A decrease of this type should be considerable at K-meson energies which are $\lesssim$ 50 MeV; at higher energies the cross section per nucleon in the nucleus should become comparable with the scattering cross section on free nucleons.

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*Nuclear Physics* **2** (1956/57) 281—282; *North-Holland Publishing Co., Amsterdam*

# NOTE ON THE NUMBER OF DIFFERENT TYPES OF K-MESONS

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Received 20 August 1956

The energy spectrum of $\pi$-mesons produced in $K_{\pi 3}$-meson decay apparently implies that $K_{\pi 2}(\theta)$ and $K_{\pi 3}(\tau)$ mesons are not identical particles [1]. Further experiments are required to enable one to decide whether one or two different types of K-mesons exist. Some suitable experiments have been described in ref. [2, 3].

In the present note we wish to draw attention to experiments of a different type which also serve the same purpose. Consider reactions on protons in which two K-mesons and particles of any other type are produced. The following reactions are examples:

$$\begin{aligned}
&p + p \rightarrow p + p + K^{+} + K^{-}\\
&p + p \rightarrow p + p + K^{+} + K^{-} + \pi^{0}\\
&p + p \rightarrow \Lambda + \Lambda + K^{+} + K^{+}\\
&p + p \rightarrow \Lambda + \Lambda + K^{+} + K^{+} + \pi^{+} + \pi^{-}\\
&\pi^{-} + p \rightarrow n + K^{+} + K^{-}\\
&\pi^{-} + p \rightarrow n + K^{+} + K^{-} + \pi^{0}\\
&\pi^{-} + p \rightarrow n + K^{+} + K^{-} + \pi^{+} + \pi^{-}\\
&n + p \rightarrow n + p + K^{+} + K^{-}\\
&n + p \rightarrow n + p + K^{+} + K^{-} + \pi^{+} + \pi^{-}\\
&\bar{p} + p \rightarrow K^{+} + K^{-}\\
&\bar{p} + p \rightarrow K^{+} + K^{-} + \pi^{+} + \pi^{-},
\end{aligned}$$

etc. ($\bar{p}$ is an antiproton).

Consider now the probability that the produced K-mesons decay into 2 $\pi$-mesons or into 3 $\pi$-mesons. If only one type of K-meson exists, the probability of both created K-mesons decaying into 2 $\pi$- or 3 $\pi$-mesons or of one K-meson decaying into 2 $\pi$'s and the other into 3 $\pi$-mesons will obviously be, respectively,

$$W_{\theta}^{2},\ W_{\tau}^{2},\ 2W_{\theta}W_{\tau}, \tag{2}$$

where $W_{\theta}$ is the probability of 2 $\pi$-decay and $W_{\tau}$ the probability of 3 $\pi$-decay.

If, however, $\theta$ and $\tau$ are different particles the state of affairs will be different. Let $a^2$ be the probability of formation of 2 $\theta$-mesons in any of the reactions (1) (the magnitude of $a^2$ can depend, of course, on the type of the reaction). In virtue of the symmetry of $\theta \rightarrow \tau$, $\Lambda \rightarrow \Lambda'$, $\Sigma \rightarrow \Sigma'$ [4]) the probability of production of two $\tau$-mesons in the same reaction should also be $a^2$. The probability of formation of a $\tau$-meson and a $\theta$-meson will then be $1-2a^2$.

If $W_\theta$ is the probability of $\theta$-decay in a beam which contains equal amounts of $\theta$- and $\tau$-mesons the probability of $\theta$-decay of a $\theta$-meson will be $2W_\theta$ and the probability of $\tau$-decay of a $\tau$-meson will be $2W_\tau$.

Thus, the respective probabilities that both of the K-mesons will decay into 2 $\pi$'s, or 3 $\pi$'s, or that one K-particle decays into 2 $\pi$'s and the other into 3 $\pi$'s, will be

$$4W_\theta^2 a^2,\ 4W_\tau^2 a^2,\ 4W_\tau W_\theta(1-2a^2). \tag{3}$$

Hence, excluding the case when $a^2 = \frac{1}{4}$, expressions (3) should differ from (2) and this circumstance can be exploited to determine whether one or two different types of K-mesons exist.

If two types of K-mesons exist which cannot change into each other in strong interactions, one should expect only $\theta\theta$ or $\tau\tau$ pairs to be observed in those cases when only K-mesons (but not hyperons) are involved. In this case $a^2 = \frac{1}{2}$ and (3) yields

$$2W_\theta^2,\ 2W_\tau^2,\ 0. \tag{4}$$

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*Nuclear Physics* **2** (1956/57) 283–285; *North-Holland Publishing Co., Amsterdam*

# BURST PRODUCTION BY SPIN-3/2 PARTICLES

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Received 14 September 1956

The spin of the $\mu$-meson (which was generally supposed to be a Dirac particle) has been the subject of some controversy in recent times, following a suggestion of Feynman that the masses of particles might arise entirely from the virtual interactions with other fields. Since the only strong interaction which a muon has with other particles is electromagnetic, Feynman suggested a spin 3/2 for this particle to account for its fairly large mass from the electromagnetic point of view, spin $\frac{1}{2}$ being hardly sufficient for the purpose. Mathews [1]) has recently investigated this possibility from the point of view of burst production by such particles at sea level, but finds that the Bremsstrahlung cross section for such particles is so much larger than for Dirac particles that they would give rise to many more bursts than are actually observed [2]). On the other hand, the moderately large Bremsstrahlung of the muon (assuming it to be a Dirac particle of mass 210 $m_e$ seems to be just right for accounting for the observed number of bursts, according to Christy and Kusaka [3]).

It is not surprising that the cross sections for radiative processes should increase very rapidly with the spin of the particle concerned. It is felt, however, that this evidence alone is rather inadequate for ruling out the possibility of spin 3/2 for the muon, as Mathews appears to have done, for there is yet another aspect of the problem which deserves attention, and this concerns the *fluctuation* in the number of the burst particles.

The fluctuation problem was first tackled theoretically by Furry [4]) on the basis of a greatly simplified model according to which electrons and photons are treated as like particles and ionization losses are neglected. He obtained the so-called "Furry distribution"

$$P_1(n, \bar{n}) = \frac{1}{\bar{n}}\left(1 - \frac{1}{\bar{n}}\right)^{n-1} \tag{1}$$

for the probability of producing $n$ particles, and this formed the basis of Christy and Kusaka's work. Mathews has also implicitly assumed eq. (1), for he has based his conclusions on Christy and Kusaka's result. This distribution is known to give too large a fluctuation in the number of particles and is probably valid if this number is small. On the other extreme lies the Poisson distribution

$$P_2(n, \bar{n}) = \frac{1}{n!}(\bar{n})^n e^{-\bar{n}} \tag{2}$$

which gives too small a fluctuation. The fluctuation problem has not even been partially solved for the actual cosmic-ray case and an exact solution seems to be out of the question. It is safe to assume, however, that the actual fluctuation probably lies somewhere between the two extremes mentioned above.

Fortunately there exists a distribution, the so-called Polyá distribution (first mentioned in this connection by Arley [5])), viz.

$$P_3(n, \bar{n}) = \frac{1}{n!}\left(\frac{\bar{n}}{1+b\bar{n}}\right)^n [1 \cdot (1+b) \ldots (1+(n-1)b)](1+\bar{n}b)^{-\frac{1}{b}} \tag{3}$$

which reduces to the Poisson and Furry distributions in the limits $b = 0$, $b = 1$, respectively. This distribution was applied some time ago by the author [6]) to study the effect of fluctuation of shower particles on the size of $\mu$-meson bursts, assuming the $\mu$-meson to be a Dirac particle of mass 210 $m_e$ and taking Snyder's [7]) improved data on cascade theory. The results showed that the burst-size increased fairly rapidly with the parameter $b$ (by almost a factor of 3 from $b = 0$ to $b = 1$), other things remaining the same. Agreement with the experimental data [2]) was obtained for $b = 0.87$, indicating an almost Furry-type distribution.

Now since there is as yet no direct evidence that the shower particles obey the Furry distribution (except with the drastically simple Furry model), there is nothing to prevent one from choosing a much smaller value of $b$ (close to zero). This will have the effect of reducing the burst-size considerably, according to the author's conclusions [6]). This reduction can then be "compensated" by a large increase in the Bremsstrahlung cross section if a spin 3/2 is assumed for the $\mu$-meson. However, the possibility of "over-compensation" is small, since Mathews' numerical estimates of the relative magnitudes of the Bremsstrahlung cross sections for spin 3/2 and spin ½ particles are likely to be considerably modified if *radiation damping* for spin 3/2 particles is considered. (Radiation damping is expected to decrease Mathews' value considerably, since its importance increases with the spin of the particle.)

It may be remarked that the purpose of this note is not to suggest a 3/2 spin for the $\mu$-meson, but only to point out that unless something definite is known about the fluctuation of the shower particles, it is not possible to give a clear-cut verdict on the $\mu$-meson spin (if the conclusion has to be based on burst production data alone.)

A detailed paper based on the author's calculations is expected to be completed shortly and will be published later.

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# NEWS ITEMS

## SYMPOSIUM ON THE PRECISION DETERMINATION OF NUCLEAR MASSES IN MAINZ (10-12 July 1956)

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Received 4 October 1956

The German mass-spectroscopist Professor J. Mattauch, present director of the ,,Max Planck Institut für Chemie" in Mainz, celebrated a few months ago his sixtieth birthday. This "calamity", as he described it, was commemorated by an international symposium on the precise determination of nuclear masses. Almost all important centres in the world working on mass spectroscopy were represented, as well as important groups in the field of microwave mass determination, nuclear reaction measurements and beta spectroscopy. This symposium, which was organized under the able direction of Professor H. Hintenberger, presented therefore a very complete view of the problems in nuclear mass determination.

Professor O. Hahn, president of the Max Planck Gesellschaft, gave the opening address. Thereafter, Bainbridge reviewed the present status of the determination of nuclear masses. Details were discussed by Endt (nuclear masses in the S—Sc region from $Q$-value measurements), Mattauch (describing among other things a very complete least-square computation of masses of light nucleides), Nier (atomic masses of stable nucleides $10 \leqq A \leqq 30$) and Van Patter (comparison of masses obtained from mass spectroscopy and nuclear reaction data). The discussions showed that at present the mass determinations by mass spectroscopy become more and more reliable. Only a few years ago discrepancies were apparent of a few tens of keV between mass values determined by mass spectroscopy as compared to those derived from nuclear reactions for masses below 40. Nuclear reaction energies measured by different groups agreed well, whereas the data obtained by mass spectroscopy showed inconsistencies. Consequently, nuclear masses were computed from reaction energies alone. The recent work of Smith *et al.* with their mass synchrometer, and even more that of Nier and his collaborators now yields, however, apparently much more precise and dependable mass doublet measurements. The discrepancies with the nuclear reaction data are now reduced to the order of magnitude of the errors assigned to the reaction values; remaining differences may at least partly be due to the compounding of errors of the reaction energy measurements used in the computation. The consensus of opinion at the conference was that mass spectrometry will probably yield the most reliable absolute mass values, whereas nuclear reaction energy measurements offer the most dependable results for mass differences.

In connection with the foregoing, Duckworth suggested to select a few nucleides ($S^{32}$, $Ti^{48}$, $Zn^{64}$, $Kr^{84}$ or $^{86}$, $Pd^{104}$, $Sn^{120}$, $Xe^{132}$, $Nd^{144}$, $W^{186}$ or $^{188}$, $Hg^{202}$, $Th^{232}$) as atomic mass substandards; these should be measured by mass-spectroscopic methods with high precision. Other, neighbouring nucleides can then be derived by nuclear reaction measurements.

Bainbridge, Duckworth, Hintenberger, Nier and Ogata presented descriptions of new mass spectrometers, either completed recently or under construction. It was impressive to note the great care used in the construction of these instruments, and their large dimensions compared with those of older types. Ewald (the stigmatic focussing mass spectrograph) and Higatsberger (electrostatic time of flight mass spectrometer) described spectrometers based on new principles. Duckworth and

Nier reported new measurements in several parts of the atomic table. Geschwindt reported on microwave mass measurements. Buechner and Van Patter discussed recent nuclear reaction energy measurements, and Mitchell commented on results obtained from beta spectroscopy.

The discussions of these lectures and that of Wapstra (the precision of the determination of spectral line positions) stressed the fact, already known to insiders, that mass spectroscopists use a different policy in assigning errors to their results than workers in the other fields mentioned above. In comparing data, it is therefore advisable to multiply errors given by mass spectroscopists by nearly a factor three. The data given both by mass spectroscopists and by nuclear reaction groups probably contain systematic errors which are not small compared with the assigned errors. Consequently, one should be cautious in using the probability law for error propagation — and, therefore, also least-square methods — in combining different experimental data.

Regularities in the nuclear binding energies obtained from the experimental data were discussed by Cappeler and Miss Way. Wapstra proposed a correction to the empirical mass equation of Green and Engler.

The lectures and discussions will be published by Pergamon Press Ltd., London.

The limited number of persons admitted to this very nicely organized conference made possible a very intensive discussion. The social program was of high quality. For both reasons the participants felt indeed as pleased as was expressed in the closing addresses.

## INTERNATIONAL CONGRESS OF THEORETICAL PHYSICS
## SEATTLE, 17-22 September 1956

At the initiative of the University of Washington, Seattle, an international congress was convened with the aim of surveying some of the great problems of topical interest in theoretical physics. The programme and organisation of the debates were extremely well-planned and the conference was one of the most successful of the numerous gatherings of physicists which have taken place in the last few years. There were about 250 participants, among whom 148 invited guests from the United States and 18 other countries, the largest single contingent being a group of 25 Japanese.

The topics covered by the congress will appear from the enumeration of the salient points of the programme:

(1) Theory of relativity by V. Bargmann (Princeton)
(2) Structure of nuclei by V. F. Weisskopf (M.I.T.)
    The nuclear shell model by J. H. D. Jensen (Heidelberg)
    The collective excitations of nuclei by B. Mottelson (CERN)
    Nuclear reactions by A. M. Lane (Harwell)
        Discussion group on nuclear models (J. A. Wheeler, Princeton)
        Discussion group on nuclear reactions (L. Rosenfeld, Manchester)
(3) The problems of statistical thermodynamics by L. van Hove (Utrecht)
    Discussion panel on statistical thermodynamics, presided by G. E. Uhlenbeck (Ann Arbor)
(4) Theory of the solid state by C. Kittel (Univ. of California)
    Superfluidity and superconductivity by R. P. Feynman (Cal. Tech.)
        Discussion group on the solid state (A. Abragam, Saclay)
        Discussion group on low temperature physics (R. P. Feynmann, Cal. Tech.)
(5) Discussion panel on quantum field theory, presided by J. R. Oppenheimer (Princeton)
        Discussion group on field theory (T. D. Lee, Columbia Univ.)

(6) Meson theories by H. Yukawa (Kyoto)
Pi-mesodynamics by F. E. Low (Univ. of Illinois)
Weak interactions by L. Michel (Lille)
New particles by C. N. Yang (Princeton)
Discussion group on meson theory (H. P. Lehmann, CERN)
(7) Stellar energy sources by E. E. Salpeter (Cornell Univ.)
Origin of cosmic rays by P. Morrison (Cornell Univ.)

Comment in this journal must be confined to the topics falling within its scope. So far as nuclear physics is concerned, the picture which emerged from the talks and discussions could not be very different from that presented by the Amsterdam Conference in July. Quantum field theory gave the general impression, perhaps not unexpected, of being bogged in formalism. It would indeed have made a comparatively dull show, had it not been for the important contribution by Prof. Bogolyubov, who presented a general theory of the dispersion relations.

On the subject of elementary particles, Prof. Schwinger, following up a suggestive analogy between electrodynamics and pi-mesodynamics, proposed a new classification of hyperons and mesons, based on the concept of hypercharge, by which the couplings with the K-meson field are characterised.

The last session aptly illustrated the bearing of nuclear physics upon cosmological problems. Recent progress in this field includes a considerable extension of the building-up process of elements. The crucial point here is the efficiency of the process by which $C^{12}$ can be built up out of 3 $\alpha$-particles; from $^{12}C$ upwards successive captures of $\alpha$-particles and neutrons account for the production of the various elements up to iron with the observed abundances.

L. R.

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